Problem Set 4

MA 111 Spring 2010

Give complete and thorough answers to these problems on separate sheets of paper. The assignment is due in class on March 31.

Problem A: In class we defined a function F which determines whether or not a certain configuration of the 15-puzzle is solvable. We proved that if F has a value of -1 then the configuration is not solvable. Give a careful, thorough, and complete explanation of why that is the case. In other words, explain our proof in your own words.

Problem B: You are handed a sliding block puzzle with numbered blocks in the following pattern. The lower right hand space does not have a block. You are asked to solve the puzzle. Explain why this cannot be done.

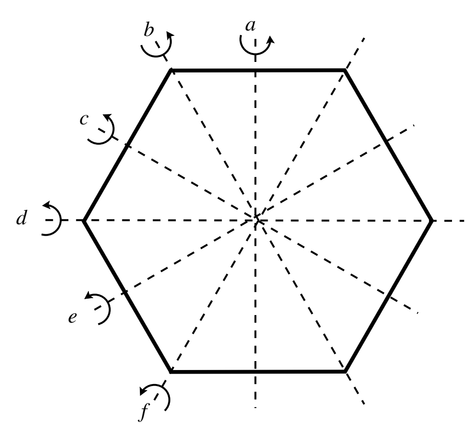
1	2	3	4
5	6	7	8
12	9	10	11
13	14	15	

Problem C: Explain why it is impossible to decorate a regular decagon so that the decorated decagon has exactly 8 symmetries.

Problem D: Recall that D_6 consists of the symmetries of a regular hexagon. Label the reflections as in the picture. The rotational symmetries are $\mathbf{I}$, R_{60}, R_{120}, R_{180}, R_{240}, R_{300}. Consider the subgroup

$$H = \{\mathbf{I}, R_{120}, R_{240}\}$$

List all the cosets of H in D_6.



Problem E: Think of $g = [1 \to 2 \to 3 \to] \circ [4 \to 5 \to 6 \to]$ as a symmetry in $\mathbb{S}_6$. Let $H = \langle g \rangle$.

(1) How many symmetries are in H?
(2) Explain why g is in A_6.
(3) Explain why H is a subgroup of both A_6 and $\mathbb{S}_6$.
(4) How many distinct cosets of H in $\mathbb{S}_6$ are there?
(5) How many distinct cosets of H in $\mathbb{A}_6$ are there?