

Problem Set 3

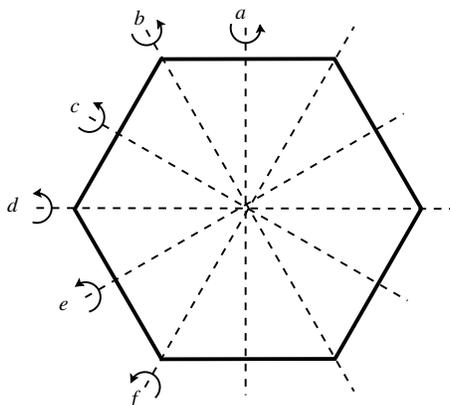
MA 111 Spring 2009

Complete the following problems on a separate sheet of paper. This assignment is due **Wednesday, March 11**.

Problem 1: Recall that D_6 consists of the symmetries of a regular hexagon. Label the reflections as in the picture. The rotational symmetries are \mathbf{I} , R_{60} , R_{120} , R_{180} , R_{240} , R_{300} . Consider the subgroup

$$H = \{\mathbf{I}, R_{120}, R_{240}\}$$

List all the cosets of H in G .



Problem 2: Suppose that p is a prime number and that G is a group containing p symmetries.

- (1) If H is a subgroup of G , how many symmetries might be in H ?
- (2) Explain why G can be generated by any symmetry other than \mathbf{I} . (Hint: Let g be a symmetry and consider the subgroup $\langle g \rangle$.)

Problem 3: Think of $g = [1 \rightarrow 2 \rightarrow 3 \rightarrow] \circ [4 \rightarrow 5 \rightarrow 6 \rightarrow]$ as a symmetry in \mathbb{S}_6 . Let $H = \langle g \rangle$.

- (1) How many symmetries are in H ?
- (2) Explain why g is in A_6 .
- (3) Explain why H is a subgroup of both A_6 and \mathbb{S}_6 .
- (4) Calculate $[\mathbb{S}_6 : H]$.
- (5) Calculate $[A_6 : H]$.

Problem 4: How many symmetries does a dodecahedron have? Be sure to carefully explain why this is the case.