MA 122: Weekly HW 3

Answer these questions on a separate sheet of paper. Remember that your work must be very neat and complete.

Problem 1: Let (a_i) be a sequence such that each a_i is one of the following numbers:

$$\{0,1,2,3,4,5,6,7,8,9\}$$

(a) Prove that the series

$$\sum_{i=1}^{\infty} \frac{a_i}{10^i}$$

converges. (Hint: Use the comparison test.)

(b) Explain the relevance of (a) to explaining why there exist numbers with an infinite number of digits to the right of the decimal point.

Problem 2: Consider the series

$$1+3x+9x^2+27x^3+81x^4+\ldots$$

(a) Use the obvious pattern to write the series in summation notation.
(b) Explain why the series is a geometric series.
(c) For what values of x does the series converge?

Problem 3: Determine whether or not the following series converge or diverge. If a sequence has both positive and negative terms also determine if the sequence is absolutely convergent.

(a) $\sum_{n=1}^{\infty} \frac{3}{n+2}$
(b) $\sum_{n=1}^{\infty} \frac{3}{\sqrt{3+n}}$
(c) $\sum_{n=1}^{\infty} \frac{\ln(n)}{n}$
(d) $\sum_{n=1}^{\infty} \frac{1}{n^2+2n+2}$
(e) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$
(f) $\sum_{n=2}^{\infty} \frac{3^n}{(n-2)!}$
(g) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{3+n}}$
(h) $\sum_{i=0}^{\infty} \frac{(-1)^i \pi^{2i}}{(2i)!}$

Problem 4: Find the radius and interval of convergence for each of the following power series.

(a) $\sum_{i=0}^{\infty} \frac{x^{2i}}{(2i)!}$
(b) $\sum_{n=0}^{\infty} nx^n$
(c) $\sum_{n=0}^{\infty} (n!)x^n$
(d) $\sum_{n=0}^{\infty} 5^n x^n$

Problem 5: For each of the following functions, find its Maclaurin series and determine the radius and interval of convergence.

(a) $f(x) = \frac{1}{1-x}$.
(b) $f(x) = e^{x^2}$. (Hint: substitute x^2 into the Maclaurin series for e^x.)

Problem 6: Let $f(x) = |x-2|$. Recall that this means that

$$f(x) = \begin{cases} -(x-2) & x \leq 2 \\ (x-2) & x \geq 2 \end{cases}$$

(a) Find a formula for the nth MacLaurin polynomial for $f(x)$.
(b) Find an upperbound (in terms of x) for the absolute value of the error $E_n(x)$ of the nth MacLaurin polynomial of $f(x)$ in terms of x for $x \geq 0$.
(c) For what values of x does the $|E_n(x)| \to 0$ as $n \to \infty$? What happens for the other values of x?
(d) What is the MacLaurin series for $f(x)$?
(e) What is the interval of convergence of the MacLaurin series for $f(x)$?
(f) Write a few sentences discussing the significance of the results of (c) and (e).