Math 439 Fall 2024

Topics in Analysis:

Measure Theory and Fourier Analysis

MWF 11:00-1:50PM Davis 217

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"I have to pay a certain sum, which I have collected in my pocket. I take the bills and coins out of my pocket and give them to the creditor in the order I find them until I have reached the total sum. This is the Riemann integral. But I can proceed differently. After I have taken all the money out of my pocket I order the bills and coins according to identical values and then I pay the several heaps one after the other to the creditor. This is my integral." – Henri Lebesgue

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Course Description: This semester we will explore two beautiful and powerful areas of mathematics: The theory of measure and integration and Fourier analysis. We will begin our semester studying measure and integration theory and it will be our primary focus for roughly two-thirds of the semester. While there are many ways to approach this subject, including a historical top-down and hands-dirty approach, we shall instead build the subject from the bottom up, treating it axiomatically. The advantages of this approach are numerous and, in particular, it immediately allows one to talk about probability theory from a rigorous perspective as it was developed by Kolmogorov and his contemporaries. In our study of measure and integration theory, we will establish important convergence theorems of Fatou, Levi, and Lebesgue (and later Vitali). We'll use our results to study important Banach and Hilbert spaces (called the Lebesgue spaces) which are used to understand the physical world and, in particular, the quantum mechanical world. We will then study decomposition, representation, an generation of measures, á la theorems of Carathéodory, Hahn, Jordan, Lebesgue, Nikodým, and Radon. After this, we shall turn our attention to Fourier analysis. We will use or sophisticated techniques of integration and measure to develop the Fourier transform and see a number of its remarkable properties and applications.

Prerequisites: Mathematics 338.

Textbook: *The Elements of Integration and Lebesgue Measure* by Robert G. Bartle, Wiley Classics Library, 1995[1]

[1]There is a 1966 edition that will also suffice.

Supplementary Course Material: When learning a subject, it is often helpful to have several resources available to compare, contrast and get different perspectives as you process material. To this end, you may want to look at the following resources: *Real Analysis* by E. Stein and R. Shakarchi and *Real Analysis: Modern Techniques and Their Applications* by G. Folland.

Grading: Your grade will be calculated as follows:

Weekly homework assignments:	35%
Presentations:	40%
Final project:	25%

Homework: Homework is an important part of this course. It is where you will grapple with new ideas, come up with creative solutions and communicate your thoughts and understanding to others. Consequently, it is crucial that you take homework very seriously. You should start homework early and work diligently. If you are having substantial difficulty with a particular exercise, please come talk to me. I am here to help! You are permitted and encouraged to discuss homework with your classmates, however, when it comes time to write up your solutions, you are required to do so independently and away from your peers (in another room, preferably). You are also permitted to consult other textbooks and, in this case, please give full details (beyond what's in our textbook). If you work with peers or consult material outside the textbook, you are required to cite your sources (including naming the peers with whom you worked); failure to do so is a matter of academic dishonesty and will result in grade penalties and/or reports of academic negligence/dishonesty. Your solutions should communicate your individual process and understanding of the material. What you turn in must be your own.

I expect homework solutions to be written out correctly and presented in good mathematical prose. Your grade will depend on the correctness of your solutions and the quality of your writing. This means that your writing should follow a coherent logical structure which makes use of complete sentences and follows standard rules of grammar. Please do not submit solutions containing incoherent and unstructured calculations. You should be proud of the material you turn in!

Homework Structure and Schedule: Assignments will be posted to the course website and are to be submitted in class on their due date. It is crucial to keep up with the homework in this course. Therefore, short of the circumstances discussed in the attendance policy above, late homework will not be not accepted. I will drop your lowest homework score on the condition that you attend one department colloquium and write a short (one paragraph) summary.

Homework Policies:

1. All write-ups are to be submitted in hard copy and should be single-sided and stapled.
2. You are encouraged to typeset your write-ups (and paper for the project) in LaTeX(I can help you set this up if you are interested and unfamiliar with LaTeX). It is however completely acceptable to submit handwritten write-ups provide your handwriting is clear and legible.
3. Each paper you turn in should have your name and the due date printed clearly at the top.
4. Your write-ups should be stapled. It is your responsibility to make sure that your homework is complete and all pages are accounted for.

5. If you used technology in the solution of the problem, please provide the complete printout showing all code, inputs and outputs in sequence, and annotate the printout with comments explaining what is being done in each step.

Presentation: This is a seminar-style course and thus all students will present material regularly. Generally speaking, I will lecture on the course material on Mondays and students will be assigned course material to present (well ahead of time) on Wednesdays and Fridays. To make things a bit more tractable (and for you to have someone to bounce ideas off of about course material), presentations will be given to students in pairs. Each pair of students will meet regularly to divide presentation time/material as well as practice/hone their presentations ahead of class. I understand that presenting mathematics is a difficult skill and takes a while to get good at; however, it is arguably the best way to learn any material: to be able to teach it yourself. As we progress through the semester, I will work with all of you individually and help you develop your presentation skills (as well as answering any questions you have about your presentations and the material before your scheduled time). I am confident that this will be a very rewarding experience for you and I am looking forward to working with you as you learn this subject and develop your presentation skills.

Final project: An important part of this course is a project to be completed by the end of the semester. The primary goal of the project is to give you an opportunity to demonstrate what you've learned throughout the semester and to experiment with new ideas beyond what we've covered in class. Toward the middle of the semester, we will meet to discuss and select a project aligned with your interests. Though the nature (and specifics) of each project will differ from student to student, the projects will culminate in a paper and presentation.

Attendance and Class Participation: As regular presentations and class participation is a large part of this course, on-time attendance is **required**. It is essential that you are present not only for your own presentation, but also to support your peers during their presentations. If something outside of your control comes up and you cannot make it to class, please let me know as soon as possible. Absence for religious reasons or for official Colby activities are excused. Unexcused absences will affect your final grade.

Office Hours & Open Door Policy The practices of learning and doing mathematics benefit from interacting with others – discussing it, asking questions, and explaining it to others – my office hours (and "open door hours") are designed for exactly this.
I maintain an "open door policy" meaning that, if you find my (inner) office door open and I'm inside, I am likely available to chat and help. In that case, please knock on my door and ask if I am available. My schedule is posted outside my office door and, more or less, lists the time you can expect me to be around and have open door hours.
Whether you make use of office hours or my "open door" policy, please interact with me (and your peers) frequently. Be ready to ask questions, exchange ideas, and be challenged. In this way, we all learn together!//
My email policy: I love talking about mathematics and I always prefer to do it in person. For this reason, I like to, whenever possible, reserve email for logistical things. However, when you are stuck and cannot come see me in person, please feel free to email me. I am here to help!

As my life is busy and I have many responsibilities, I usually only check and respond to email once per day and sometimes not at all on weekends. For this reason, I try to uphold the following

24-48 hour rule: If you send an email Sunday through Thursday, I will do my best to respond within 24 hours. If you send an email on Friday or Saturday, I will do my best to respond within 48 hours. If I do not respond within these windows, feel free to email me again as I may have missed it.

Academic Integrity: Honesty, integrity, and personal responsibility are cornerstones of a Colby education and provide the foundation for scholarly inquiry, intellectual discourse, and an open and welcoming campus community. These values are articulated in the Colby Affirmation and are central to this course. You are expected to demonstrate academic honesty in all aspects of this course. If you are clear about course expectations, give credit to those whose work you rely on, and submit your best work, you are highly unlikely to commit an act of academic dishonesty.

Academic dishonesty includes, but is not limited to: violating clearly stated rules for taking an exam or completing homework; plagiarism (including material from sources without a citation and quotation marks around any borrowed words); claiming another's work or a modification of another's work as one's own; buying or attempting to buy papers or projects for a course; fabricating information or citations; knowingly assisting others in acts of academic dishonesty; misrepresentations to faculty within the context of a course; and submitting the same work, including an essay that you wrote, in more than one course without the permission of the instructors.

Academic dishonesty is a serious offense against the college. Sanctions for academic dishonesty are assigned by an academic review board and may include failure on the assignment, failure in the course, or suspension or expulsion from the College.

For more on recognizing and avoiding plagiarism, see libguides.colby.edu/avoidingplagiarism

The Colby Affirmation

Colby College is a community dedicated to learning and committed to the growth and well-being of all its members.

As a community devoted to intellectual growth, we value academic integrity. We agree to take ownership of our academic work, to submit only work that is our own, to fully acknowledge the research and ideas of others in our work, and to abide by the instructions and regulations governing academic work established by the faculty.

As a community built on respect for ourselves, each other, and our physical environment, we recognize the diversity of people that have gathered here and that genuine inclusivity requires active, honest, and compassionate engagement with one another. We agree to respect each other, to honor community expectations, and to comply with college policies.

As a member of this community, I pledge to hold myself and others accountable to these values.