As promised, below is an itemized list of topics we've covered in Math 381 since the second midterm exam. As you prepare for the final exam, you should place this list of topics together with the previous two to form a comprehensive list. As I stated in class, the final exam will be comprehensive but it will have a strong focus on post-midterm 2 material (approximately 60-75% of the final will focus on material from after the second midterm). In studying for the final exam, please note that I consider the homework exercises (including both "turn in" and "do not turn in" problems) and the examples I've covered in lecture to be the best source of practice problems. If you know how to approach each problem, are able to work quickly and accurately, and understand the theory and methodology by which you have obtained a solution, you should perform well on the exam.

Theory

You should know all definitions precisely. In particular, you should know the definitions of the following terms and be able to list several examples/applications for each:

1. What it means for a random variable to be continuous (i.e., that it have a density (Definition 5.31)).
2. What a density function is (Definition 5.28).
3. In addition to the discrete random variables we discussed for midterm 2, you know know (by heart) the following random variables, what they model, their densities, and their mean and variance:
   (a) Uniform $X \sim Unif([a,b])$.
   (b) Exponential $X \sim Exp(\lambda)$ for $\lambda > 0$.
   (c) Normal $X \sim \mathcal{N}(\mu, \sigma)$ with mean μ and variance σ^2).
4. You should know how the expectation of a continuous random variable is defined (via its density). This is theorem 5.41.
5. For any random variable X, you should know the definition of X's associated cumulative distribution function F_X and how to use it to determine if a random variable is discrete or continuous.
6. You should know what it means for a pair of random variables X and Y to be jointly discrete (i.e., they are two discrete random variables defined on the same sample space Ω and you should know the definition of their joint probability mass function, $p_{X,Y}$. Given jointly discrete random variables, you should know the definition (and how to compute) the marginals p_X and p_Y.
7. You should know what it means for a pair of random variables X and Y to be jointly continuous including the definition of their joint probability density function, $f_{X,Y}$, and joint cumulative distribution function $F_{X,Y}$. Also, you should know the definition of the marginals f_X and f_Y.
8. You should know what it means for two (or more) random variables X and Y to be independent. (See Definitions 6.13 and 6.16 in the course notes.)
9. You should know the idea of joint expectation for two random variables X and Y and a function $\varphi(X,Y)$ on a countable sample space. Note: This isn't really different from our initial definition of expectation.
10. You should know the definition of covariance for two random variables X and Y.
11. You should know the definition of the convolution product.
12. You should know the definition of conditional expectation $E(X|Y=y)$ for each $y \in R(Y)$ (Definition 6.29) and, also, the random variable $E(X|Y)$ (Definition 6.30)
13. You should know the three types of convergence given in Definition 7.1.
14. You should know the definition of the moment generating function $M_X(t)$ associated to a random variable X.

We have discussed many properties and results in this course (concerning the concepts listed above). For the final exam, you should know, in particular, the following results (properties derived, theorems, propositions, etc.). Note: I have indicated (with an asterisk "$*$") which properties you should be able to show/argue in detail. This, in general, does not mean prove but you should have a VERY solid idea of how things are gotten and which properties are used and when.

* Lemma 5.34
* Lemma 5.37
* Corollary 5.38
* Proposition 5.40 (Note – you proved this in Exercise 5.18 and you should know that exercise well).
- Theorem 5.45
* Proposition 5.47 (Note: The ideas behind the proof align exactly with the procedure done in the Pizzas! and the Laser Beam example. You should have a very good understanding of this procedure).
* Proposition 6.2
* Proposition 6.5
- Proposition 6.7
* Proposition 6.8 (it is a quick consequence of Proposition 6.7 and the axioms of probability)
* Proposition 6.12
* Proposition 6.14
* Proposition 6.15
- Theorem 6.17
* Proposition 6.19
* Proposition 6.20
* Corollary 6.21
* Proposition 6.23
* Proposition 6.24
* Proposition 6.25
* Proposition 6.27 (you should have an idea of how the proof works for the discrete random variable case which we did in class).
* Theorem 6.31 (you should know, at least, the proof in the discrete case).
- Proposition 7.2
* Theorem 7.3 (You should know the proof of the Markov and Chebyshev inequalities too)
- Theorem 7.6
- Theorem 7.7
* Proposition 7.9
- Proposition 7.10
* Proposition 7.11
* Corollary 7.12

Computations and Problem Solving

The following are computations that you should know how to do (accurately and quickly).

1. As stated before, you must know all of the major random variables (and their mass/density functions by heart and know how to work with all of them).
2. If I gave you a density function $f_X(x)$ of a continuous random variable, you should be able to compute probabilities and expectations associated to that random variable.
3. If I give you a density function $f_X(x)$ of a continuous random variable, you should be able to compute its cumulative distribution function.
4. If I give you a function F, you should be able to verify whether or not it is the cumulative distribution function of some random variable. If it is the CDF of some random variable, i.e., $F = F_X$, you should be able to tell whether or not the random variable is a continuous random variable and, if it is, you should be able to compute its density.
5. If I give you a continuous random variable X and a function $\varphi : \mathbb{R} \to \mathbb{R}$, you should be able to tell if $\varphi(X)$ is continuous also and, if it is, compute its density.
6. You should be able to work examples like Pizza! and Laser beam without trouble (and so know how to obtain a density function from another view the CDF).
7. If I give you a function $p(x, y)$, you should be able to determine if it is the joint probability mass function of two random variables X and Y.
8. If $p_{X,Y}(x, y)$ is the joint probability mass function of two random variable, you should be able to compute the marginal probability mass functions p_X and p_Y. You should also be able to tell whether or not X and Y are independent.
9. If I give you a reasonable function $f(x, y)$, you should be able to determine if it is the probability density function of jointly continuous random variables X and Y. If it is, i.e., $f = f_{X,Y}$, you should be able to compute their marginal densities. Also, you should be able to tell if X and Y are independent.
10. For jointly discrete or jointly continuous random variables, you should be able to compute lots of probabilities (either through double summation or double integration) of events defined in terms of these random variables. In particular, you should have a solid command of the material in the examples and exercises in Sections 6.1 and 6.2 of the notes. This includes being able to compute certain double integrals using polar coordinates.
11. You should be able to do computations involving expectations of jointly discrete or jointly continuous random variables X and Y. To this end, you should have a solid command of all of the material in Section 6.1 including all examples and exercises. This includes being able to compute certain double integrals using polar coordinates.
12. If I give you two independent random variables and their probability mass function (or density functions), you should be able to compute the probability mass function (or density function) of their sum via Proposition 6.26.
13. Given jointly discrete or jointly continuous random variables mass or density functions $p_{X,Y}$ or $f_{X,Y}$, respectively, you should be able to compute their conditional mass or density functions, $p_{X|Y}$ or $f_{X|Y}$ and use them to compute conditional probabilities (as in Examples 6.15-6.18).
14. Given two independent random variables X and Y, you should be able to compute the distributions of their sum $X + Y$ via convolution (via Proposition 6.27).
15. Given two random variables (say jointly discrete or continuous), you should be able to compute the conditional expectation $E(X|Y = y)$ and verify the law of iterated expectation.

16. For a random variable X (discrete or continuous), you should be able to compute its moment generating function.

17. You should be able to use a moment generating function to compute moments of a random variable.

18. Given a random variable X and a sequence of random variables $X_1, X_2, \dots$, you should be able to determine if the sequence converges in distribution to X using their moment generating functions (i.e., using Proposition 7.13).

19. You should be able to use convergence in distribution to approximate probabilities. For example, if I know that a sequence Y_n of random variables converges in distribution to a uniform random variables $Y \sim U(0,1)$, then
$$P(1/4 \leq Y_n \leq 1/2)$$
is approximately
$$P(1/4 \leq Y \leq 1/2) = \int_{1/4}^{1/2} 1\, dx = \frac{1}{4}$$
where the approximation gets better and better as n becomes larger. Do you see why this follows from knowing that they converge in distribution? Could you do it for other random variables Y?

You should be familiar and comfortable with most every example we have done in lecture and covered in the course notes (and homework). This list includes but is not limited to:

1. Examples with continuous random variables and jointly continuous random variables. In particular, you should know everything about all of the major examples we've studied throughout the course including: Pizzas!, Lasers, Alice's random walk, joint discrete die roll, marble, dartboard, two customers.

2. All examples of moment generating functions (and using the results to compute moments).

Special Notes and some things not to worry about

1. You should read the convergence examples, 7.3-7.6. It's good practice for understanding these notions of convergence and how they are different. I won't ask you to recall the calculations on these examples, but you should try to understand what's going on.

2. You won't be asked about the proof of the central limit theorem nor the proof of the strong law of large numbers. Though you should understand the statements of both and have an idea what they are saying.