As promised, I have written an itemized list of topics we've covered in Math 338 since the previous midterm exam. As I stated in class, the exam will cover Chapter 4 of Rudin and Chapter 6 of our supplementary course notes. This list of topics (and the proportion of time we've spend on them since the end of Midterm 1 material) will align with the problems you will see on Wednesday's midterm exam. In studying for Wednesday's midterm exam, please note that I consider the homework exercises and the everything I've covered in lecture to be the best source of practice (problems, proofs, etc). If you know how to approach each problem/exercise/proof, are able to work quickly and accurately, and understand the theory and methodology by which you have obtained a solution/proof, you should perform well on the exam.

Definitions:

The following list enumerates all the definitions you need to know by heart. In particular, you should make sure to know all quantifiers involved in the definitions and the order in which they appear. Also, for each definition, you should be able to come up with several examples satisfying the definition (and hopefully things that don't satisfy the definition).

1. For a function $f : X \to Y$ where X and Y are metric spaces, you should know Definition 4.1 which quantifies what it means for $f(x) \to q$ as $x \to p$ (equivalently $\lim_{x\to p} f(x) = q$. Note: This is the $\epsilon - \delta$ definition.
2. For a function $f : X \to Y$, you should know what it means for f to be continuous at a point p in X This is definition 4.5. As we discussed in class, the δ can depend on *both* ϵ and p.
3. You should know what it means for a function $f : X \to \mathbb{R}^d$ to be bounded (this is definition 4.13).
4. You should know what it means for a function $f : X \to Y$ to be uniformly continuous on X. You should know that this differs from continuity in that the δs involved can only depend on ϵ.
5. For a function f whose domain is an interval (a, b) in $\mathbb{R}$. Given a point x, you should know how the left and right limits of f are defined. It should be noted that my notation is slightly different from Rudin – you are free to use whatever you'd like.
6. For a function f whose domain is an open interval (a, b) and which has a discontinuiuty at $x \in (a, b)$, you should know what it means for this discontinuity to be simple (of the first kind) and also what it means for the discontinuity to be of the second kind.
7. You should know what it means for a function $f : (a, b) \to \mathbb{R}$ to be monotonic (and the finer distinction between monotonically increasing and monotonically decreasing).
8. Please take a look at the sections on limits at infinite and infinite limits. I won't ask that you recall these for the exam, but you should know about them.
9. For a compact interval $I = [a, b] \subseteq \mathbb{R}$, you should know what it means for a set $P = \{x_0, x_2, \dots, x_K\}$ to be a partition of I. You should also know that, for each k, $\Delta x_k = x_k - x_{k-1}$, and the norm/size of the partition is defined as $\|P\| = \max_{k=1,2,\dots,K} \Delta x_k$.
10. Given a partition P of I as above, you should know that it means for a set $\{x_1^*, x_2^*, \dots, x_K^*\}$ to be admissible for P.
11. For a bounded function f on I, i.e., $f \in B(I)$, and a partition P of I, you should know the definition of upper and lower Darboux sums with respect to P. These are, respectively,

$$U(f, P) = \sum_{k=1}^{K} M_k \Delta_k$$

and

$$L(f, P) = \sum_{k=1}^{K} m_k \Delta_k$$

where

$$M_k = \sup_{x_{k-1}\leq x\leq x_k} f(x) \qquad \text{and} \qquad m_k = \sup_{x_{k-1}\leq x\leq x_k} f(x)$$

for $k = 1, 2, \ldots, K$.

12. You should know that the upper and lower Riemann-Darboux sums (also called integrals) are defined by

$$U(f) = \inf_P U(f, P) \qquad \text{and} \qquad L(f) = \sup_P L(f, P)$$

where we note that these infima/suprema are taken over the set of all partitions P of I.

13. You should know that it means for $f \in B(I)$ to be Riemann/Darboux integrable on I. Of course, this happens exactly when $U(f) = L(f)$.

14. You should know that a Riemann sum for $f \in B(I)$ is, by definition, a sum of the form

$$\sum_{k=1}^{K} f(x_k^*)\Delta x_k$$

where $\{x_1^*, x_2^*, \ldots, x_K^*\}$ is admissible for a given partition P of I.

15. You should know the definition of the signed integral $\int_b^a f$ when $f \in R([a, b])$.

16. You should know what it means for a bounded complex-valued function of a real variable $x \in I = [a, b]$ to be Riemann/Darboux integrable, i.e., $f \in R(I; \mathbb{C})$ if and only if $u = \mathrm{Re}(f), u = \mathrm{Im}(f) \in R(I; \mathbb{R})$. In this case, the Riemann/Darboux integral of f is

$$\int_a^b f(x)\,dx = \int_a^b u(x) + i\int_a^b v(x)\,dx$$

Results (Theorems, propositions, lemmas, corollaries):

For the following results, unless otherwise mentioned, you should know the statement of the result precisely and have a really good idea of how they are proved – ideally, you should be able to reproduce the proof. This is especially true of all named theorems. Note: Some of my proofs given in class differ from those in Rudin. It doesn't matter which proof you know/understand – either is fine.

1. You should know the characterization of limits in terms of sequences (Theorem 4.2). You should also know its corollary – limits are unique.

2. You should know the results concerning the algebra of limits (Theorem 4.4) for real-valued functions. Note, this follows from the analogous proof of the algebra of limits for sequences and you should have a good understanding of how that proof works too.

3. You should know Theorem 4.6 and the argument I gave in class (which is a little more detailed than that of Rudin).

4. You should know that the composition of continuous functions is continuous and how to prove it. Rudin does this in Theorem 4.7 and I did this in class using the characterization of continuous functions in terms of open sets (using Theorem 4.8). While Rudin's result is slightly more general, knowing either argument is fine.

5. You should know (and understand the proof extremely well) of Theorem 4.8. You should also know its corollary (in terms of closed sets).

6. You should know the result of Theorem 4.9 (but don't worry, I won't ask you to prove it).

7. You should know Theorem 4.14 and its proof.
8. You should know Theorems 4.15 and Theorem 4.16 (the EVT) and how they are derived from Theorem 4.14.
9. You should know Theorem 4.19 and have a very very good idea of how its proof works. This is truly a powerful (and beautiful) argument.
10. You should be familiar with the examples following Theorem 4.20 (the the result), but I certainly won't ask you about the proof in detail.
11. You should know Theorem 4.23 (the IVT) and its proof.
12. You should know the examples of the section on Discontinuities. While we didn't talk about this in class, you should take a look at the Thomae function (which appears in Rudin's exercises and has substantial treatment in Abbott – it's an amazing example!)
13. You should be able to state and prove Theorem 4.29. Note: I stated it as a lemma in class.
14. You should know the corollary to Theorem 4.29 and also the resulting theorem, Theorem 4.30.
15. You should know Lemma 6.3 and its proof.
16. You should know Lemma 6.4 and its proof.
17. You should know Proposition 6.5 and its proof (which proceeds it).
18. You should know Theorem 6.7 and its proof.
19. You should know Theorem 6.8 and have a very good idea of how the proof works. Note that it uses in a fundamental way the fact that continuous functions on compact sets are uniformly continuous.
20. You should know Corollaries 6.9 and 6.10 and their easy proofs.
21. You should know Theorem 6.12 and why it is powerful. As I discussed in class it gives a way to use the abstract construction (which is elegant though difficult to compute) to say something useful about Riemann sums (which are inelegant but easy to compute). You do not need to know more of the proof than I covered in class.
22. You should know the corollary to Theorem 6.12 (Corollary 6.13) and how you might implement it, say, for a continuous function on $[0, 1]$. Note: This is probably how you did some computations in your very first calculus course.
23. You should know Theorem 6.14 and its proof.
24. You should know Proposition 6.15 and its proof.
25. You should know Proposition 6.16. While its proof is easy and not terribly difficult to remember, I don't find it all that crucial. It's just a simple observation.
26. You should know Theorem 6.18 While its proof is easy and not too difficult to write down, there is nothing deep going on so don't spend much time on this.
27. You should know Theorems 6.20 and 6.21. As their proofs are simple corollaries to things we've already worked hard on, don't worry about their proofs.
28. You should know Theorem 6.22 (the triangle inequality) and its clever proof. I'll talk about it on monday.
29. You should know Lemma 6.23 and its proof.
30. You should read and be familiar with the results of Section 6.4, but I won't ask any direct questions on this material on Wednesday's exam (this will be on the final). We will likely cover this material on Wednesday.