As promised, I have written an itemized list of topics we've covered in Math 338 since the previous midterm exam. As I stated in class, the exam will cover Chapter 4, some of Chapter 5, and some of Chapter 9 in Rudin's book, Principles of Mathematical Analysis[1]. This list of topics (and the proportion of time we've spend on them since the end of Midterm 1 material) will align with the problems you will see on Wednesday's midterm exam. In studying for Wednesday's midterm exam, please note that I consider the homework exercises and the everything I've covered in lecture to be the best source of practice (problems, proofs, etc). If you know how to approach each problem/exercise/proof, are able to work quickly and accurately, and understand the theory and methodology by which you have obtained a solution/proof, you should perform well on the exam.

Definitions:

The following list enumerates all the definitions you need to know by heart. In particular, you should make sure to know all quantifiers involved in the definitions and the order in which they appear. Also, for each definition, you should be able to come up with several examples satisfying the definition (and hopefully things that don't satisfy the definition).

1. For a function $f : X \to Y$ where X and Y are metric spaces, you should know Definition 4.1 which quantifies what it means for $f(x) \to q$ as $x \to p$ (equivalently $\lim_{x\to p} f(x) = q$. Note: This is the $\epsilon - \delta$ definition.

2. For a function $f : X \to Y$, you should know what it means for f to be continuous at a point p in X This is definition 4.5. As we discussed in class, the δ can depend on *both* ϵ and p.

3. You should know what it means for a function $f : X \to \mathbb{R}^d$ to be bounded (this is definition 4.13).

4. You should know what it means for a function $f : X \to Y$ to be uniformly continuous on X. You should know that this differs from continuity in that the δs involved can only depend on ϵ.

5. For a function f whose domain is an interval (a, b) in $\mathbb{R}$. Given a point x, you should know how the left and right limits of f are defined. It should be noted that my notation is slightly different from Rudin – you are free to use whatever you'd like.

6. For a function f whose domain is an open interval (a, b) and which has a discontinuiuty at $x \in (a, b)$, you should know what it means for this discontinuity to be simple (of the first kind) and also what it means for the discontinuity to be of the second kind.

7. You should know what it means for a function $f : (a, b) \to \mathbb{R}$ to be monotonic (and the finer distinction between monotonically increasing and monotonically decreasing).

8. Please take a look at the sections on limits at infinite and infinite limits. I won't ask that you recall these for the exam, but you should know about them.

9. You should know the definition of differentiability for a real-valued function f defined on an interval $[a, b]$.

10. For a function f defined on an open set $\mathcal{O} \subseteq \mathbb{R}^n$ and mapping into $\mathbb{R}^m$, you should know what it means for f to be differentiable at a point $\mathbf{x}_0 \in \mathcal{O}$. You should also know what the derivative means and thoroughly understand its interpretation as the matrix that yields the "best" affine approximation to the function at the point $\mathbf{x}_0$.

Results (Theorems, propositions, lemmas, corollaries):

For the following results, unless otherwise mentioned, you should know the statement of the result precisely and have a really good idea of how they are proved – ideally, you should be able to reproduce the proof. This is especially true of all named theorems. Note: Some of my proofs given in class differ from those in Rudin. It doesn't matter which proof you know/understand – either is fine.

[1] Of course, you should be reading Abbott simultaneously (his book has amazing explanations and he gives great intuition), but we work primarily from Rudin and the list below reflects this.

1. You should know the characterization of limits in terms of sequences (Theorem 4.2). You should also know its corollary – limits are unique.
2. You should know the results concerning the algebra of limits (Theorem 4.4) for real-valued functions. Note, this follows from the analogous proof of the algebra of limits for sequences and you should have a good understanding of how that proof works too.
3. You should know Theorem 4.6 and the argument I gave in class (which is a little more detailed than that of Rudin).
4. You should know that the composition of continuous functions is continuous and how to prove it (Theorem 4.7). Challenge: Could you use the characterization of continuity in terms of open sets to prove it too – if you knew that both functions were continuous everywhere?
5. You should know (and understand the proof extremely well) of Theorem 4.8. You should also know its corollary (in terms of closed sets).
6. You should know the result of Theorem 4.9 (but don't worry, I won't ask you to prove it).
7. You should know Theorem 4.14 and its proof.
8. You should know Theorems 4.15 and Theorem 4.16 (the EVT) and how they are derived from Theorem 4.14.
9. You should know Theorem 4.19 and have a very very good idea of how its proof works. This is truly a powerful (and beautiful) argument.
10. You should be familiar with the examples following Theorem 4.20 (the the result), but I certainly won't ask you about the proof in detail.
11. You should know Theorem 4.23 (the IVT) and its proof.
12. You should know the examples of the section on Discontinuities. You should also know the details of Thomae's function (though Rudin only mentions it in the exercises).
13. You should be able to state and prove Theorem 4.29. Note: I stated it as a lemma in class.
14. You should know the corollary to Theorem 4.29 and also the resulting theorem, Theorem 4.30.
15. You should know Theorem 5.2 (and also think about analogous versions for differentiable functions mapping from $\mathbb{R}^n$ to $\mathbb{R}^m$.
16. You should have some idea of how Theorem 5.3 is proved. (and definitely know the result).
17. You should know the chain rule (Theorem 5.5) – and, at least, the proof that's called for in Exercise 2 in Homework 7. I also (will) prove the chain rule for multivariate functions in class on Monday and you should have a very solid understanding of the result and its proof. Note: This latter result appears as Theorem 9.15 in Rudin.

Additional Concepts from Lecture and Homework Stuff

As we've only had two homeworks since the preceding midterm, I expect you to know the exercises on these two homeworks (and their solutions) backwards and forwards. To serve this goal, I am happy to write solutions to these homeworks; the call for solution requests ends Monday night (5:00PM), April 15th. Also, if you have any questions about the homeworks, please come and ask me. Beyond the things appearing directly in the homework, I spent time in class on Friday April 12 and (will spend) Monday April 15 discussing various topics from multivariate analysis (also going by the names: (rigorous) vector calculus, analysis on Manifolds, etc.). In particular, you should know the following.

In what follows, unless otherwise mentioned, $\mathcal{O} \subseteq \mathbb{R}^n$ is an open set and $f : \mathcal{O} \to \mathbb{R}^m$. For this function and in terms of the Euclidean norm/metric on the domain and codomain, you should know:

1. The definition of continuity for f at $\mathbf{x}_0 \in \mathcal{O}$.
2. The definition of differentiability for f at $\mathbf{x}_0$.
3. In the case of differentiability, you should know what its derivative is (and how it is computed).
4. You should know how to show that a function is differentiable given a candidate for its derivative (matrix).
5. You should know how the notion of differentiability (stated as in Homework 7) implies continuity.
6. You should know all the results in homework 7 concerning the uniqueness of derivatives (and affine approximations). Also, you should know, in the case of differentiability, why the derivative matrix is formed by the entries of f's partial derivatives.
7. You should know the algebra of derivatives for such multivariate functions.
8. You should know the chain rule for multivariate functions, how its proved and used.

Things not to worry about:

1. The proof of Theorem 4.10 (but the result is good to know and Rudin goes through good and useful examples following the theorem).
2. Don't worry about knowing the proof to Theorem 4.17. You should be familiar with what the statement says but I won't directly ask you about it.
3. Don't worry about Theorem 4.34.
4. For this exam, you don't need to know any of the things in Chapter 5 of Rudin from Page 107 (mean value theorems) until the end of the chapter.