As promised, I have written an itemized list of topics we've covered in Math 338 since the beginning of the semester. As I stated in class, the exam will cover Chapters 1, 2, and 3 of Rudin's book, Principles of Mathematical Analysis[1]. This list of topics (and the proportion of time we've spend on them since the beginning of the semester) will align with the problems you will see on Wednesday's midterm exam. In studying for Wednesday's midterm exam, please note that I consider the homework exercises and the everything I've covered in lecture to be the best source of practice (problems, proofs, etc). If you know how to approach each problem/exercise/proof, are able to work quickly and accurately, and understand the theory and methodology by which you have obtained a solution/proof, you should perform well on the exam.

Definitions:

The following list enumerates all the definitions you need to know by heart. In particular, you should make sure to know all quantifiers involved in the definitions and the order in which they appear. Also, for each definition, you should be able to come up with several examples satisfying the definition (and hopefully things that don't satisfy the definition).

1. Ordered set.
2. In an ordered set S, what it means for a subset E to be bounded above (or below).
3. For an ordered set S and a subset E, you should know what it means for β to be an upper (or lower) bound of E.
4. For an ordered set S an a subset E which is bounded above (or below), you should know the definition of least upper bound/supremum (or greatest lower bound/infimum) of E.
5. Least upper bound property.
6. Definition of Field
7. Ordered field.
8. You have a solid understanding of what $\mathbb{R}$ is (see Theorem 1.19). You should also understand the definition of the extended real numbers (Definition 1.23).
9. You should know the definition of the complex numbers $\mathbb{C}$ and how they relate to $\mathbb{R}$. You should, in particular, understand the two representations of complex numbers we discussed: $z = (a, b) = a+ib$ and the multiplication structure for such numbers that makes them into a field.
10. Definition 1.30 and 1.32.
11. Definition 1.36 (Note, I will always use $\mathbb{R}^d$ instead of Rudin's $\mathbf{R}^k$).
12. Definitions. 2.1-2.4
13. Example 2.4 – note: The conventions we will use (which are distinct from Rudin) are:

$$\mathbb{N} = \{0, 1, 2, \dots\}, \qquad \mathbb{N}_+ = \{1, 2, \dots, \}, \text{ and } \qquad \mathbb{Z} = \{0, \pm 1, \pm 2, \pm 3, \dots\}.$$

14. Definition 2.7
15. You should know the concept of looking at a collection (either finite or infinite (countable or uncountable) of sets. This is essentially Definition 2.9, but I find Rudin's discussion somewhat confusing. Just know things at the level of 274 and what we've done in class.

[1]Of course, you should be reading Abbott simultaneously (his book has amazing explanations and he gives great intuition), but we work primarily from Rudin and the list below reflects this.

16. Definition of metric space.
17. Definition 2.18 (I'm happy if you use the term accumulation point instead of limit point)
18. Definition of closure, Definition 2.26
19. You should know the definition of compact (Definitions 2.31 and 2.32).
20. You should know the construction of the Cantor set, especially in the amount of detail we looked at it in the homework.
21. You should know the definition of separated and connected.
22. Definition of convergence in a metric space, Definition 3.1.
23. Definition 3.5
24. Definition 3.8 – you should really!!!!! understand this.
25. You should know the definition of diameter of a set in a metric space.
26. You should know the definition of "complete metric space".
27. Definition 3.13
28. Definition 3.15 (this one of more of a remark/convention)
29. Definition 3.16
30. Definition 3.21 (in my view, this definition goes almost the whole page 59) and we took a lot of time discussing it in class – you need to know every aspect of it.
31. Definition of e.
32. Definition 3.38
33. The definition of absolute convergence and conditional convergence for a series σa_n.

Results (Theorems, propositions, lemmas, corollaries):

For the following results, unless otherwise mentioned, you should know the statement of the result precisely and have a really good idea of how they are proved – ideally, you should be able to reproduce the proof. This is especially true of all named theorems. Note: Some of my proofs given in class differ from those in Rudin. It doesn't matter which proofs you know/understand – either is fine.

1. Theorem 1.11
2. Proposition 1.14
3. Proposition 1.15
4. Proposition 1.16
5. Proposition 1.18
6. Theorem 1.19 (You do not need to know the proof of this)
7. Theorem 1.20
8. Theorem 1.21

9. Corollary to theorem 1.21
10. Theorem 1.25-1.29
11. Theorem 1.31
12. Theorem 1.33
13. Theorem 1.35 (the Cauchy-Schwarz inequality)
14. Theorem 1.37
15. You should know that $\mathbb{N}$, $\mathbb{N}_+$ and $\mathbb{Z}$ are all countable.
16. Theorem 2.8
17. De Morgan's laws (you don't need to know how prove these – at least for this course)–these appear as Theorem 2.22
18. Theorem 2.12 and its corollary (You don't need to know hot to prove these – at least for this course)
19. Theorem 2.13-2.14 (You don't need to know how to prove these – at least for this course).
20. Theorem 2.19
21. Theorem 2.20 and its corollary
22. Theorem 2.23 and its corollary
23. Theorem 2.24
24. Theorem 2.27
25. Theorem 2.28
26. Theorem 2.34
27. Theorem 2.35 and its corollary
28. Theorem 2.36
29. Theorem 2.38
30. Theorem 2.39
31. Theorem 2.40
32. Theorem 2.41 (note this contains the statement of the Heine-Borel theorem)
33. Theorem 2.42
34. Theorem 2.43 and its immediate corollary.
35. Theorem 2.47 (you won't be asked to reproduce its proof, but you should definitely know the result).
36. Theorem 3.2.
37. Theorem 3.3.
38. Theorem 3.4
39. At the bottom of page 51, Rudin says: It is clear that $\{p_n\}$ converges to p if and only if every subsequence of $\{p_n\}$ converges to p. We leave the details to the reader. – You should know these details.

40. Theorem 3.6 – Note: this statement contains the Bolzano-Weierstrass theorem.
41. Theorem 3.10 – Note: You proved part of this (before we saw it in the book) in your homework.
42. Theorem 3.11 – this one is super important!
43. Theorem 3.14
44. All results concerning lim sups and lim infs. This includes Theorem 3.17 but, in fact, we did MUCH better in the homework because we proved a construction of the lim sup that you can actually use (and we have!).
45. Theorem 3.19
46. Theorem 3.20
47. Theorem 3.22 (The so-called Cauchy criterion for series).
48. Theorem 3.23
49. Theorem 3.24
50. Theorem 3.25 (and it's finer statement involving absolute convergence which we discuss on Monday).
51. Theorem 3.26 – on Geometric series.
52. Theorem 3.27 (Cauchy condensation)
53. Theorem 3.28
54. Theorem 3.29
55. Theorem 3.31
56. Theorem 3.33 – the Root test (and refinements for absolute convergence)
57. Theorem 3.34 – the Ratio test (and refinements for absolute convergence)
58. Theorem 3.39 – radius of convergence for power series via the root test. You should also know an analogous test for power series which gives radius of convergence via the ratio test (and how to prove it).
59. Theorem 3.41 and Theorem 3.42 – at least at the level that you did it in Homework 5.
60. Theorem 3.43 – alternating series test
61. Theorem 3.44
62. Theorem 3.45
63. Theorem 3.47

Things not to worry about:

1. Don't worry too much about 1.22 (you should know that every real number has a decimal expansion).
2. Don't worry about anything in the appendix of Chapter 1 (at least, for the exam you don't need to worry about it. If you plan to continue studies of analysis going forward, you should definitely be familiar with this appendix).
3. Don't worry about "open relative to", e.g., Theorem 2.30 and 2.33. I won't ask you about this directly on this exam. If you study topology beyond this course, you should learn it.

4. You should know the result of Theorem 3.32 but you don't need to know its proof.
5. You should know the result of Theorem 3.37 but you don't need to know its proof. Don't worry too much about Theorems 3.50, 3.51 (or definition 3.48).
6. You don't need to know the proofs of the results of rearrangements (but you should know how absolutely amazing the results are!!!!)