As I discussed on the last day of class, the final will be cumulative and the problems that you will see on it will be taken from a sample that is distributed uniformly from all the topics we considered throughout the semester. In particular, you will not see more than one question focused on the material following Midterm 2. You can expect to a little less than half of the exam focused on the topics before Midterm 1 and the same amount focused on the topics between Midterms 1 and 2. In this list of topics, I will focus only on the few things we have done following the second midterm and so, as you prepare for your final exam, I encourage you to put all of the lists of topics together to form one big list. In studying for the final exam, please note that I consider the homework exercises and the examples I've covered in lecture to be the best source of practice problems. If you know how to approach each problem, are able to work quickly and accurately, and understand the theory and methodology by which you have obtained a solution, you should perform well on the exam.

Theory

1. You should know the definition of autonomous (often non-linear) $n \times n$ system. Of course, this is a first-order system of ordinary differential equations of the form

$$\dot{\mathbf{x}} = F(\mathbf{x})$$

   where F is generally a function from $\mathbb{R}^n$ into itself.

2. An equilibrium point for the system above is, by definition, an element $\mathbf{x}_0 \in \mathbb{R}^n$ for which $F(\mathbf{x}_0) = 0$. Of course, to each equilibrium point there corresponds an equilibrium solution which is constant: $\mathbf{x}(t) := \mathbf{x}_0$ for all t.

3. You should know what it means for equilibrium solutions to the above system to be sinks, sources, nodes. Further, in the 2×2 case, you should know what is meant by spiral sink/source/center.

4. Provided that F is differentiable, you should know the theory surrounding the solutions to the system near equilibrium points. In short, we studied that Hartman-Grobman theorem which says that the behavior of solutions of $\dot{\mathbf{x}} = F(\mathbf{x})$ near an equilibrium point $\mathbf{x}_0$ will behave similarly to solutions to the constant coefficient equation

$$\dot{\mathbf{y}} = A\mathbf{y}$$

   where A is the $n \times n$ matrix

$$A = DF(\mathbf{x}_0) = \begin{pmatrix} \frac{\partial F_1}{\partial x_1} & \frac{\partial F_1}{\partial x_2} & \cdots & \frac{\partial F_1}{\partial x_n} \\ \frac{\partial F_2}{\partial x_1} & \frac{\partial F_2}{\partial x_2} & \cdots & \frac{\partial F_2}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial F_n}{\partial x_1} & \frac{\partial F_n}{\partial x_2} & \cdots & \frac{\partial F_n}{\partial x_n} \end{pmatrix}$$

   where all partial derivatives of the components of F are evaluated at $\mathbf{x}_0$. In particular, one can classify the nature of the equilibrium solution (source, sink, node) in terms of the eigenvalues of the matrix $DF(\mathbf{x}_0)$. Note: I won't give you anything worse than 3×3 (i.e., $n = 3$).

Procedures/Solution Methods

1. For an autonomous system ($n = 2$ or $n = 3$) of the form $\dot{\mathbf{x}} = F(\mathbf{x})$, you should be able to:
   (a) Find equilibrium points/solutions.
   (b) At an equilibrium point, calculate $DF(\mathbf{x}_0)$ and find its eigenvalues/vectors.
   (c) Use what you found to classify the equilibrium solutions as sinks, sources, nodes. Also, be able to discuss the nature of the behavior of nearby solutions qualitatively.

   In the above, you should know Examples 3 and 4 in Section 4.4.

As the course notes are a bit light on the above subject, I encourage you to see Sections 5.1 and 5.2 of "Differential Equations by Blanchard, Devaney, and Hall.