MATH 253: PROJECT III

DIAGONALIZATION

This final project for Math 253 covers diagonalization. The purpose of this project is to learn the basics of diagonalization, well enough to be able to teach them to someone else. To this end, you will thoroughly learn the material in Sections 5.2 and 5.3 in our textbook – including all definitions, theorems, proofs, and examples. Once you learn this material, please complete one of the following two options:

Option 1 Design and record a (15-30 minute) video lecture covering Section 5.3 in our textbook. To this end, you must

(a) For each example (Examples 1-6), come up with your own original example which is different from the corresponding textbook example, but conveys the same message and illustrates the same things (See Section 2 below). You should present your six examples in the same order as the textbook examples (interleaved with the theorems), but your examples must be original and different in numbers from the textbook examples.

(b) Present Theorem 5 and its proof. Your presentation of the theorem should be completely precise. Your proof should convey to me that you fully understand what's going on.

(c) Present Theorem 6 and its proof. Your presentation of the theorem should be completely precise. Your proof should convey to me that you fully understand what's going on.

(d) Present Theorem 7. Your presentation of the theorem should be completely precise.

You may use any video platform you want to record. In fact, Zoom has a recording feature that works well. If you choose this option, you should submit your video as a .mp4 file. I would like your lectures to be presented in real time (as my lectures are). Please don't use Keynote, Powerpoint, etc. I would like you to write out the examples and theorems as you talk the viewer through them (similar to my lectures). Of course, I don't expect you to have a chalk/white board at home. Thus, you may write on any surface you have available in a way that is legible to the viewer (taped up paper, refrigerator, etc. – get creative!). I will be the only person watching (and grading) these lectures.

Option 2 Write a thorough summary of Sections 5.2 and 5.3 of high expository quality. In doing this, please meet the following requirements:

(a) You will need to present all theorems and proofs which appear in Sections 5.2 and 5.3. Though your theorems can be stated in the same words as the textbook (though, in every case, you must cite the result in the textbook), the proofs must be presented in **your original words**, written in clear English and presented in good mathematical prose. The proofs should demonstrate that you fully understand the result and why it is true. Simply rearranging and modifying the words from the textbook will not suffice – the proofs need to be written in your own words and convey your understanding.

(b) For each example in the textbook, you will design a corresponding example (which is different in numbers) but illustrates the same things that the textbook example does (See Section 2 below).

(c) This is an expository summary. It should be well-written with clear prose, correct grammar and punctuation, with a minimal use of unnecessary symbols[1], and VERY few typos. Beyond formal mathematical statements taken from the textbook[2] – which I expect to be properly cited and credited – everything you write should be in your own words using your own voice. Your summary should be **between 8 and 15 pages and typesetted in LaTeX** and your final submission should be compiled (from LaTeX) as a PDF. To make things easier for you (and for me), I have created a template in Overleaf that you should use (available here). You can use any LaTeX editor/compiler of your choosing, e.g., Overleaf, Texmaker, TeXstudio, Emacs, etc. If you need help getting started with LaTeX, please let me know and I'm happy to help.

1. Big Details:

(1) Whichever option you choose, your submission is due on **May 18th by 11:59PM** (in your respective time zone).
(2) If you choose Item 1, you must submit an .mp4 file including your video lecture covering Section 5.3. If you choose Item 2, you must submit an expository write-up on diagonalization which covers Sections 5.2 and 5.3. as a PDF file which has been typeset in LaTeX.
(3) In completing this project, please keep in mind good practices of academic integrity. For this, I encourage you to see http://www.colby.edu/academicintegrity/
(4) As you work on this project, I **strongly** encourage you to reach out for help. We can discuss things via email or, if you'd like, I'm happy to schedule a Zoom meeting with you.
(5) No matter what you do, your result should convey that you fully understand the material.

2. What do I mean by "come up with your own original examples"?

Let's suppose that, for the purpose of this project, you are being asked to come up with an original example which is analogous to Example 2 on Page 303 of our textbook. A satisfactory analogous example (which is original to me) is the following:

Example 1

Let
$$A = \begin{pmatrix} 1 & 1 \\ 3 & -1 \end{pmatrix}, \quad \mathbf{u} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad \text{and} \quad \mathbf{v} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}.$$
Are $\mathbf{u}$ and $\mathbf{v}$ eigenvectors for A?

Solution: We have
$$A\mathbf{u} = \begin{pmatrix} 1 & 1 \\ 3 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix} = 2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} = 2\mathbf{u}$$
and so $\mathbf{u}$ is an eignvector of A with eigenvalue 2. Now
$$A\mathbf{v} = \begin{pmatrix} 1 & 1 \\ 3 & -1 \end{pmatrix} \begin{pmatrix} 3 \\ -1 \end{pmatrix} = \begin{pmatrix} 2 \\ 10 \end{pmatrix} \neq \lambda \begin{pmatrix} 3 \\ -1 \end{pmatrix}$$
for any $\lambda \in \mathbb{R}$ and hence $\mathbf{v}$ is not an eigenvector for A.

[1] Do not use "$\implies$", "$\exists$", "$\nexists$", "$\forall$", "$\therefore$"
[2] This means definitions, theorems, etc.