This review sheet should be a guide to the *new* major topics and concepts that will be tested on the final exam. All content from the previous two midterm review sheets may also be on the final! For practice problems, please revisit your homework assignments or complete additional exercises from the textbook.

Gradient stuff (Chapters 14.4-14.6)

You should be able to:

1. Explain what it means for a function $f(x, y)$ to be differentiable at a point (x_0, y_0).
2. Compute the linear approximation $T_1(x, y)$ of a function $f(x, y)$ at a given point.
3. Find an equation for the tangent plane to a given surface S at a point (x_0, y_0, z_0). This surface S could be the graph of a function $f(x, y)$ (which is differentiable at (x_0, y_0)) or defined implicitly by $g(x, y, z) = c$ (where g is differentiable at (x_0, y_0, z_0) and c is a constant).
4. State and apply the chain rule for functions of two or three variables.
5. Compute and interpret the directional derivative of a function f in the direction of a vector $\mathbf{u}$.
6. For a differentiable function f, use the gradient to find the direction of greatest increase (or decrease) of a function f at a given point. In general, you should have a good understanding of what the gradient tells us. So, if we have a function f with non-zero gradient, moving in the direction $\mathbf{u} = \nabla f / \|\nabla f\|$ will move us in the direction of greatest increase. Also, in what direction would you move to keep f constant?
7. Understand the orthogonality relationship between the gradient and the level curves of a function $f(x, y)$.

Optimization (Chapter 14.7)

For a function $f(x, y)$ which is differentiable, you should be able to

1. Solve for the critical points of a function f on the interior of a region.
2. In the case the f has continuous second partial derivatives, classify critical points of $f(x, y)$ as local extrema or saddle points using the second derivative test.
3. Identify maxima and minima of a function f on the boundary of a region.
4. Find global minima or maxima, if they exist, of a function f.
5. In doing this, you should also think about what you would do if the function $f(x, y)$ fails to be differentiable (or even continuous) on the region. For example, if a function f is not continuous on a closed and bounded region, must it have a global maximum/minimum? What if the function is continuous but not differentiable?

Double integrals (Chapters 15.1-15.4)

You should be able to:

1. Define the double integral of a function $f(x, y)$ on a 2D rectangle $\mathcal{R} = [a, b] \times [c, d]$ as the limit of a Riemann sum.
2. State and apply Fubini's theorem relating double integrals and iterated integrals over rectangles. In this, you should be able to compute double integrals for sufficiently nice functions over a rectangular region.

3. For a region $D \subseteq \mathbb{R}^2$, you should know what it means to integrate a function f over D. You should also recognize
$$\iint_D f(x,y)\,dA$$
as a "signed" volume under the graph of f. In the case that $f(x,y) = 1$, this is how we define the area of D, i.e.,
$$\text{Area}(D) = \iint_D 1\,dA.$$

4. You should also know the basic properties of integrals over regions. For example the integral is "linear" in the sense that, for functions f and g that are integrable over D,
$$\iint_D f(x,y) + g(x,y)\,dA = \iint_D f(x,y)\,dA + \iint_D g(x,y)\,dA$$
and, for any constant C,
$$\iint_D Cf(x,y)\,dA = C\iint_D f(x,y)\,dA.$$
Also, if f and g are integrable over D and $f(x,y) \leq g(x,y)$, must it be true that
$$\iint_D f(x,y)\,dA \leq \iint_D g(x,y)\,dA?$$

5. Given a region $D \subset \mathbb{R}^2$, you should be able to identify if it is a "Type 1" or "Type 2" region (in the vocabulary of Stewart). If it is a Type 1 region, i.e., it has clear "bottom" and "top" functions, you should be able to identify these functions g_1 and g_2 and then express D as
$$D = \{(x,y) | a \leq x \leq b,\, g_1(x) \leq y \leq g_2(x)\}.$$
You should also be able to do everything above (identifying h_1 and h_2) in the case D is a Type 2 region.

6. Using the results we obtained for Type 1 and Type 2 regions, you should be able to compute the double integral of f over D (see, e.g., Theorems 3 and 5 in Section 15.3).

7. For a region D that is both Type 1 and Type 2, you should be able to convert a double integral of the form
$$\int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)\,dy\,dx$$
into an integral of the form
$$\int_c^d \int_{h_1(y)}^{h_2(y)} f(x,y)\,dx\,dy.$$
In other words, you should be able to switch the order of integration between Type 1 and Type 2 regions. Recall, sometimes these computations are easier one way than the other.

8. Use abstract properties of double integrals (additivity, monotonicity, etc.) to simplify their expressions or compute them entirely.

9. Set up and evaluate iterated integrals using polar coordinates.

10. Change the coordinates of an iterated integral from Cartesian to polar or vice versa.