

Appendix 1: Data Section

Table 27.6.1: Ground State (GS) Spectroscopic Properties of Diatomic Molecules.²⁻⁷

Molecule	$\tilde{\nu}_e(\text{cm}^{-1})$	$\tilde{\nu}_e x_e(\text{cm}^{-1})$	$\tilde{B}_e(\text{cm}^{-1})$	$\tilde{\alpha}_e(\text{cm}^{-1})$	$\tilde{D}_e(\text{cm}^{-1})$	$R_e(\text{\AA})$	$D_0(\text{eV})$	GS
$^1\text{H}^1\text{H}$	4401.21	121.34	60.853	3.062	0.0471	0.7414	4.4776	$^1\Sigma_g^+$
$^7\text{Li}^1\text{H}$	1405.498	23.168	7.51373	0.21639	8.617×10^{-4}	1.5957	2.429	$^1\Sigma_g^+$
$^{12}\text{C}^1\text{H}$	2860.75	64.44	14.460	0.536	14.5×10^{-4}	1.1199	3.46	$^2\Pi$
$^{14}\text{N}^1\text{H}$	3282.7	79.0	16.6679	0.6504	17.1×10^{-4}	1.0362	3.69	$^3\Sigma^-$
$^1\text{H}^{19}\text{F}$	4138.385	89.943	20.9537	0.7934	21.51×10^{-4}	0.91681	5.86	$^1\Sigma_g^+$
$^1\text{H}^{35}\text{Cl}$	2990.925	52.800	10.5933	0.3070	5.32×10^{-4}	1.27455	4.432	$^1\Sigma_g^+$
$^7\text{Li}^7\text{Li}$	351.407	2.583	0.67253	0.00705	9.87×10^{-6}	2.6729	1.06	$^1\Sigma_g^+$
$^{12}\text{C}^{16}\text{O}$	2169.756	13.288	1.93160	0.01751	6.121×10^{-6}	1.1283	11.108	$^1\Sigma_g^+$
$^{14}\text{N}^{14}\text{N}$	2358.6	14.324	1.99824	0.01732	5.8×10^{-6}	1.0977	9.756	$^1\Sigma_g^+$
$^{14}\text{N}^{16}\text{O}$	1904.135	14.088	1.70489	0.01754	0.54×10^{-6}	1.15077	6.497	$^2\Pi$
$^{16}\text{O}^{16}\text{O}$	1580.16	11.9513	1.4456	0.01593	4.839×10^{-6}	1.20752	5.126	$^3\Sigma_g^-$
$^{35}\text{Cl}^{35}\text{Cl}$	559.75	2.6943	0.2442	0.00152	0.186×10^{-6}	1.988	2.476	$^1\Sigma_g^+$

Table 27.8.2: Character Tables for Common Point Groups.

$C_s=C_h$	E	σ_h	h = 2	
A'	1	1	x, y, x^2 , y^2 , z^2 , xy	R_z
A''	1	-1	z, yz, xz	R_x , R_y
$\Gamma_{x,y,z}$	3	1		

$C_i=S_2$	E	i	h = 2	
A_g	1	1	x^2 , y^2 , z^2 , xy, xz, yz	R_x , R_y , R_z
A_u	1	-1	x, y, z	
$\Gamma_{x,y,z}$	3	-3		

C_{2v}	E	C_2	σ_v	σ_v'	h = 4	
A_1	1	1	1	1	z, z^2 , x^2 , y^2	
A_2	1	1	-1	-1	xy	R_z
B_1	1	-1	1	-1	y, yz	R_x
B_2	1	-1	-1	1	x, xz	R_y
$\Gamma_{x,y,z}$	3	-1	1	1		

C_{3v}	E	$2C_3$	$3\sigma_v$	h = 6	
A_1	1	1	1	z, z^2 , x^2+y^2	
A_2	1	1	-1		R_z
E	2	-1	0	(x,y), (x^2-y^2 , xy), (xz, yz)	(R_x , R_y)
$\Gamma_{x,y,z}$	3	0	1		

C_{4v}	E	$2C_4$	C_2	$2\sigma_v$	$2\sigma_d$	h = 8	
A_1	1	1	1	1	1	z, z^2 , x^2+y^2	
A_2	1	1	1	-1	-1		R_z
B_1	1	-1	1	1	-1	x^2-y^2	
B_2	1	-1	1	-1	1	xy	
E	2	0	-2	0	0	(x,y) (xz, yz)	(R_x , R_y)
$\Gamma_{x,y,z}$	3	1	-1	1	1		

C _{5v}	E	2C ₅	2C ₅ ²	5σ _v	h = 10	α = 72°
A ₁	1	1	1	1	z, z ² , x ² +y ²	
A ₂	1	1	1	-1		R _z
E ₁	2	2 cos α	2 cos 2α	0	(x, y) (xz, yz)	(R _x , R _y)
E ₂	2	2 cos 2α	2 cos α	0	(xy, x ² -y ²)	
Γ _{x,y,z}	3	1+2 cos α	1+2 cos 2α	1		

C _{6v}	E	2C ₆	2C ₃	C ₂	3σ _v	3σ _d	h = 12
A ₁	1	1	1	1	1	1	z, z ² , x ² +y ²
A ₂	1	1	1	1	-1	-1	
B ₁	1	-1	1	-1	1	-1	R _z
B ₂	1	-1	1	-1	-1	1	
E ₁	2	1	-1	-2	0	0	(x,y) (xz, yz) (R _x , R _y)
E ₂	2	-1	-1	2	0	0	(xy, x ² -y ²)
Γ _{x,y,z}	3	2	0	-1	1	1	

D ₂	E	C ₂ (z)	C ₂ (y)	C ₂ (x)	h = 4
A ₁	1	1	1	1	x ² , y ² , z ²
B ₁	1	1	-1	-1	z, xy R _z
B ₂	1	-1	1	-1	y, xz R _y
B ₃	1	-1	-1	1	x, yz R _x
Γ _{x,y,z}	3	-1	-1	-1	

D _{2h}	E	C ₂ (z)	C ₂ (y)	C ₂ (x)	i	σ(xy)	σ(xz)	σ(yz)	
A _g	1	1	1	1	1	1	1	1	x ² , y ² , z ²
B _{1g}	1	1	-1	-1	1	1	-1	-1	xy R _z
B _{2g}	1	-1	1	-1	1	-1	1	-1	xz R _y
B _{3g}	1	-1	-1	1	1	-1	-1	1	yz R _x
A _u	1	1	1	1	-1	-1	-1	-1	
B _{1u}	1	1	-1	-1	-1	-1	1	1	z
B _{2u}	1	-1	1	-1	-1	1	-1	1	y
B _{3u}	1	-1	-1	1	-1	1	1	-1	x
Γ _{x,y,z}	3	-1	-1	-1	-3	1	1	1	

D _{3h}	E	σ _h	2C ₃	2S ₃	3C ₂ '	3σ _v	h = 12
A ₁ '	1	1	1	1	1	1	z ² , x ² +y ²
A ₂ '	1	1	1	1	-1	-1	
A ₁ "	1	-1	1	-1	1	-1	
A ₂ "	1	-1	1	-1	-1	1	z
E'	2	2	-1	-1	0	0	(x, y), (xy, x ² -y ²)
E"	2	-2	-1	1	0	0	(xz, yz) (R _x , R _y)
Γ _{x,y,z}	3	0	-1	1	-2	1	

T _d	E	8C ₃	3C ₂	6σ _d	6S ₄	h = 24
A ₁	1	1	1	1	1	x ² +y ² +z ²
A ₂	1	1	1	-1	-1	
E	2	-1	2	0	0	(3z ² -r ² , x ² -y ²)
T ₁	3	0	-1	-1	1	(R _x , R _y , R _z)
T ₂	3	0	-1	1	-1	(x, y, z), (xy, xz, yz)
Γ _{x,y,z}	3	0	-1	1	-1	

Tabel 29.1.2: ^1H Chemical Shifts of Methyl, Methylene, and Methine Groups.¹⁻³

	Methyl protons	δ_{H}	Methylene protons	δ_{H}	Methine protons	δ_{H}
C	$\text{CH}_3\text{-R}$	0.9	$\text{R-CH}_2\text{-R}$	1.4	$>\text{CH-R}$	1.5
	$\text{CH}_3\text{-C=C}$	1.1	$\text{R-CH}_2\text{-C=C}$	1.7	$>\text{CH-C=C}$	1.8
	$\text{CH}_3\text{-C-O}$	1.3	$\text{R-CH}_2\text{-C-O}$	1.9	$>\text{CH-C-O}$	2.0
	$\text{CH}_3\text{-C-N}$	1.1	$\text{R-CH}_2\text{-C-N}$	1.4	$>\text{CH-C-N}$	1.8
	$\text{CH}_3\text{-C-NO}_2$	1.6	$\text{R-CH}_2\text{-C-NO}_2$	2.1		
	$\text{CH}_3\text{-C=C}$	1.6	$\text{R-CH}_2\text{-C=C}$	2.3	$>\text{CH-C=C}$	2.6
	$\text{CH}_3\text{-Ar}$	2.3	$\text{R-CH}_2\text{-Ar}$	2.7	$>\text{CH-Ar}$	3.0
	$\text{CH}_3\text{-C=CC=O}$	2.0	$\text{R-CH}_2\text{-C=CC=O}$	2.4	$>\text{CH-C=CC=O}$	2.7
	$\text{C=C(CH}_3\text{)-C=O}$	1.8	$\text{C=C(CH}_2\text{-R)-C=O}$	2.4	C=C(CH<)-C=O	2.8
	$\text{CH}_3\text{-C}\equiv\text{C}$	1.8	$\text{R-CH}_2\text{-C}\equiv\text{C}$	2.2	$>\text{CH-C}\equiv\text{C}$	2.6
	$\text{CH}_3\text{-C(=O)-R}$	2.2	$\text{R-CH}_2\text{-C(=O)-R}$	2.4	$>\text{CH-C(=O)-R}$	2.7
	$\text{CH}_3\text{-C(=O)-Ar}$	2.6	$\text{R-CH}_2\text{-C(=O)-Ar}$	2.9	$>\text{CH-C(=O)-Ar}$	3.3
	$\text{CH}_3\text{-C(=O)-OR}$	2.0	$\text{R-CH}_2\text{-C(=O)-OR}$	2.2	$>\text{CH-C(=O)-OR}$	2.5
	$\text{CH}_3\text{-C(=O)-OAr}$	2.4	$\text{R-CH}_2\text{-C(=O)-OAr}$	2.7	$>\text{CH-C(=O)-OAr}$	2.7
	$\text{CH}_3\text{-C(=O)-N}$	2.0	$\text{R-CH}_2\text{-C(=O)-N}$	2.2	$>\text{CH-C(=O)-N}$	2.4
		2.0	$\text{R-CH}_2\text{-C}\equiv\text{N}$	2.3	$>\text{CH-C}\equiv\text{N}$	2.7
	N $\text{CH}_3\text{-N}$	2.3	$\text{R-CH}_2\text{-N}$	2.5	$>\text{CH-N}$	2.8
	$\text{CH}_3\text{-N-Ar}$	3.0	$\text{R-CH}_2\text{-N-Ar}$	3.1	$>\text{CH-N-Ar}$	3.6
	$\text{CH}_3\text{-N-C(=O)-R}$	2.9	$\text{R-CH}_2\text{-N-C(=O)-R}$	3.2	$>\text{CH-N-C(=O)-R}$	4.0
O	$\text{CH}_3\text{-N}^+$	3.3	$\text{R-CH}_2\text{-N}^+$	3.3		
			$\text{R-CH}_2\text{-NO}_2$	4.4	$>\text{CH-NO}_2$	4.7
			$\text{R-CH}_2\text{-OH}$	3.6	$>\text{CH-OH}$	3.9
	$\text{CH}_3\text{-OR}$	3.3	$\text{R-CH}_2\text{-OR}$	3.4	$>\text{CH-OR}$	3.7
	$\text{CH}_3\text{-O-C=C}$	3.8	$\text{R-CH}_2\text{-O-C=C}$	3.7	$>\text{CH-O-C=C}$	4.2
	$\text{CH}_3\text{-O-Ar}$	3.8	$\text{R-CH}_2\text{-O-Ar}$	4.3	$>\text{CH-O-Ar}$	4.5
	$\text{CH}_3\text{-O-C(=O)-R}$	3.7	$\text{R-CH}_2\text{-O-C(=O)-R}$	4.1	$>\text{CH-O-C(=O)-R}$	4.8
X			$\text{RO-CH}_2\text{-OR}$	4.8	R-O-CH-OR	4.7
			$\text{R-CH}_2\text{-F}$	4.4		
			$\text{R-CH}_2\text{-Cl}$	3.6	$>\text{CH-Cl}$	4.2
			$\text{R-CH}_2\text{-Br}$	3.5	$>\text{CH-Br}$	4.3
			$\text{R-CH}_2\text{-I}$	3.2	$>\text{CH-I}$	4.3
	$\text{CH}_3\text{-Si}$	0.0	$\text{R-CH}_2\text{-Si}$	0.5	$>\text{CH-Si}$	1.2
	$\text{CH}_3\text{-S}$	2.1	$\text{R-CH}_2\text{-S}$	2.4	$>\text{CH-S}$	3.2
	$\text{CH}_3\text{-S(=O)R}$	2.5	$\text{R-CH}_2\text{-S(=O)R}$	3.0		
	$\text{CH}_3\text{-S(O}_2\text{)R}$	2.8	$\text{R-CH}_2\text{-S(O}_2\text{)R}$	3.0		
	$\text{CH}_3\text{-S(O}_2\text{)O-R}$	3.2				
	$\text{CH}_3\text{-OS(O}_2\text{)-R}$	3.9	$\text{R-CH}_2\text{-OS(O}_2\text{)-R}$	4.3	$>\text{CH-OS(O}_2\text{)-R}$	4.7
			$\text{RS-CH}_2\text{-SR}$	3.8	RS-CH-SR	4.1

R = alkyl group. Typical uncertainty is ± 0.3 ppm for methyl and methylene and ± 0.5 ppm for methine unless electronic or anisotropic effects from other groups are strong.

1. D. H. Williams, I. Fleming, *Spectroscopic Methods in Organic Chemistry*. 4th Ed., McGraw Hill, London, UK, 1987, Table 3.17.
2. W. Hull, S. Krause, S. Kuhn, G. Schneider, C. Steinbeck, "Predict," *NMRShiftDB2*, Universität zu Köln, <http://nmrshiftdb.nmr.uni-koeln.de/>, last accessed 8/2/2016.
3. *SDBSWeb*, National Institute of Advanced Industrial Science and Technology, <http://sdbs.db.aist.go.jp>, last access 8/2/2016.

Table 29.1.4: ^{13}C Chemical Shifts of Carbonyl Carbons.¹⁻³

$\begin{array}{c} \text{O} \\ \parallel \\ \text{C} \\ / \quad \backslash \\ \text{R}_1 \quad \text{R}_2 \end{array}$			$\begin{array}{c} \text{O} \\ \parallel \\ \text{C} \\ / \quad \backslash \\ \text{R}_1 \quad \text{R}_2 \end{array}$		
R_1	R_2	δ_{C}	R_1	R_2	δ_{C}
Me-	-H	199.7	Me-	-OH	178.1
Et-	-H	206.0	Et-	-OH	180.4
Pr ⁿ -	-H	202.8	Pr ⁿ -	-OH	180.7
Pr ⁱ -	-H	204.0	Pr ⁱ -	-OH	184.1
Bu ⁿ -	-H	202.8	Bu ⁿ -	-OH	180.9
Bu ^t -	-H	205.8	Bu ^t -	-OH	185.9
CH ₂ =CH-	-H	192.4	CH ₂ =CH-	-OH	171.7
Ph-	-H	192.0	Ph-	-OH	172.6
Me-	-Me	206.0	H-	-OEt	161.4
Et-	-Me	207.6	Me-	-OMe	170.7
Pr ⁿ -	-Me	208.9	Me-	-OEt	171.1
Pr ⁱ -	-Me	211.8	Et-	-OMe	173.3
Bu ⁿ -	-Me	209.0	Pr ⁿ -	-OMe	175.7
Bu ^t -	-Me	213.5	Pr ⁱ -	-OMe	175.7
HOCH ₂ -	-Me	208.2	Bu ⁿ -	-OMe	174.2
ClCH ₂ -	-Me	200.7	Bu ^t -	-OMe	178.9
Cl ₂ CH-	-Me	193.6	CH ₂ =CH-	-OMe	165.5
Cl ₃ C-	-Me	186.3	Ph-	-OMe	166.8
CH ₂ =CH-	-Me	197.2	- (CH ₂) ₃ O -		177.9
Ph-	-Me	197.6	- (CH ₂) ₄ O -		175.2
Ph-	-Et	200.0	Me-	-NH ₂	173.4
Ph-	-Ph	196.5	Et-	-NH ₂	174.3
- (CH ₂) ₃ -		208.2	CH ₂ =CH-	-NH ₂	168.3
- (CH ₂) ₄ -		213.9	Ph-	-NH ₂	169.7
- (CH ₂) ₅ -		208.8	Ph-	-N(CH ₃) ₂	171.6
- (CH ₂) ₆ -		211.7	- (CH ₂) ₃ NH -		179.4
-CH ₂ CH ₂ CH=CH-		209.0	- (CH ₂) ₄ NH -		173.0
-CH ₂ =CH-CH=CH-		200.0	Me-	-OAc	167.3
-CH ₂ CH ₂ CH ₂ CH=CH-		198.0	Ph-	-OAc	162.8
			Me-	-Cl	170.3
			Et-	-Cl	174.7
			CH ₂ =CH-	-Cl	165.6
			Ph-	-Cl	168.3

1. D. H. Williams, I. Fleming, *Spectroscopic Methods in Organic Chemistry*. 4th Ed., McGraw Hill, London, UK, 1987, Table 3.13.
2. W. Hull, S. Krause, S. Kuhn, G. Schneider, C. Steinbeck, *NMRShiftDB2*, Universität zu Köln, <http://nmrshiftdb.nmr.uni-koeln.de/>, last accessed 8/2/2016.
3. *SDBSWeb*, National Institute of Advanced Industrial Science and Technology, <http://sdbb.db.aist.go.jp>, last accessed 8/2/2016.

Table 30.5.1: Atomic Energies in Hartrees (au) using *Ab Initio* Methods. ^{*(DS)}

Atom	HF/6-31G*	MP2/6-311G*	B3LYP/6-311G*	B3LYP/6-311+G*	B3LYP/cc-pVTZ
H	-0.4982329	-0.4998098	-0.5021559	-0.5021559	-0.5021563
Li	-7.4313723	-7.4320264	-7.4912968	-7.4913333	-7.4920159
Be	-14.5669444	-14.5985691	-14.6711842	-14.6713141	-14.6724455
B	-24.5220372	-24.5693278	-24.6618695	-24.6624669	-24.6637586
C	-37.6808603	-37.7450232	-37.8559888	-37.8572669	-37.8585746
N	-54.3854425	-54.4750512	-54.5985435	-54.6007232	-54.601781
O	-74.7839336	-74.9181455	-75.0853748	-75.0898713	-75.0918572
F	-99.3649569	-99.5541705	-99.7538101	-99.7605798	-99.7628675
Na	-161.841435	-161.845926	-162.286694	-162.286844	-162.296912
Mg	-199.5956109	-199.606556	-200.093098	-200.0932688	-200.0976386
Si	-288.8317857	-288.8921959	-289.393806	-289.3941537	-289.3963273
P	-340.6902044	-340.7671703	-341.2804859	-341.281716	-341.2858577
S	-397.4759577	-397.5807253	-398.1320492	-398.1330364	-398.1386911
Cl	-459.4479641	-459.5851374	-460.1661563	-460.1668773	-460.1746898

* SCF atom energies are identical at the “*” and “**” levels, or equivalently at the (d) and (d,p) levels.

Integral Table (a, b = constants; n, m = integers)

$$\begin{aligned} \int u \, dv &= uv - \int v \, du & \int x^n \, dx &= x^{n+1}/(n+1) & \int \ln x \, dx &= x \ln x - x \\ \int \frac{dx}{a+bx} &= \frac{1}{b} \ln(a+bx) & \int \frac{x \, dx}{a+bx} &= \frac{1}{b^2} [a+bx - a \ln(a+bx)] \\ \int \frac{dx}{x(a+bx)} &= \frac{1}{a} \ln\left(\frac{x}{a+bx}\right) & \int \frac{dx}{(a+bx)(a'+b'x)} &= \frac{1}{ab'-a'b} \ln\left(\frac{a'+b'x}{a+bx}\right) \\ \int \frac{dx}{(a+bx)^2(a'+b'x)} &= \frac{1}{ab'-a'b} \left[\frac{1}{a+bx} + \frac{b'}{ab'-a'b} \ln\left(\frac{a'+b'x}{a+bx}\right) \right] \end{aligned}$$

Gaussian Functions

$$\begin{aligned} \int_0^\infty e^{-ax^2} \, dx &= \frac{1}{2} \left(\frac{\pi}{a}\right)^{1/2} & \int_0^\infty x e^{-ax^2} \, dx &= \frac{1}{2a} \\ \int_0^\infty x^2 e^{-ax^2} \, dx &= \frac{1}{4a} \left(\frac{\pi}{a}\right)^{1/2} & \int_0^\infty x^3 e^{-ax^2} \, dx &= \frac{1}{2a^2} \\ \int_0^\infty x^4 e^{-ax^2} \, dx &= \frac{3}{8a^2} \left(\frac{\pi}{a}\right)^{1/2} & \int_0^\infty x^5 e^{-ax^2} \, dx &= \frac{1}{a^3} \\ \int_0^\infty x^{2n} e^{-ax^2} \, dx &= \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2^{n+1} a^n} \left(\frac{\pi}{a}\right)^{1/2} & \int_0^\infty x^{2n+1} e^{-ax^2} \, dx &= \frac{n!}{2} \left(\frac{1}{a^{n+1}}\right) \\ \int_0^\infty e^{-x^2/2\sigma_x^2} \, dx &= \frac{1}{2} (2\pi)^{1/2} \sigma_x & \int_0^\infty x^2 e^{-x^2/2\sigma_x^2} \, dx &= \frac{2\sigma_x^2}{4} (2\pi)^{1/2} \sigma_x \\ \frac{2}{\sqrt{\pi}} \int_0^t e^{-y^2} \, dy &= \text{erf}(t) & \frac{1}{\sqrt{2\pi}} \int_0^z e^{-x^2/2} \, dx &= \frac{1}{2} \text{erf}(z/\sqrt{2}) \\ \frac{1}{\sigma_x \sqrt{2\pi}} \int_0^{x_c} e^{-x^2/2\sigma_x^2} \, dx &= \frac{1}{2} \text{erf}\left(\frac{x_c}{\sigma_x \sqrt{2}}\right) & \int_0^{x_c} e^{-ax^2} \, dx &= \frac{\sqrt{\pi}}{2} \text{erf}(\sqrt{a} x_c) \end{aligned}$$

z	0.1	0.2	0.3	0.4	0.5	0.6	0.8	1.0	1.64485	2.0
t = z/√2	0.07071	0.14142	0.21213	0.28284	0.35355	0.42426	0.56569	0.70711	1.16309	1.41421
erf(t)	0.07966	0.15852	0.23582	0.31084	0.38292	0.45149	0.57629	0.68269	0.90000	0.95450

Exponential Functions (a > 0)

$$\begin{aligned} \int x^n e^{-ax} \, dx &= -\frac{x^n e^{-ax}}{a} + \frac{n}{a} \int x^{n-1} e^{-ax} \, dx \\ \int e^{-ax} \, dx &= -\frac{e^{-ax}}{a} \\ \int x e^{-ax} \, dx &= -\frac{x e^{-ax}}{a} - \frac{e^{-ax}}{a^2} \\ \int x^2 e^{-ax} \, dx &= -\frac{x^2 e^{-ax}}{a} - \frac{2x e^{-ax}}{a^2} - \frac{2 e^{-ax}}{a^3} \\ \int x^3 e^{-ax} \, dx &= -\frac{x^3 e^{-ax}}{a} - \frac{3x^2 e^{-ax}}{a^2} - \frac{6x e^{-ax}}{a^3} - \frac{6 e^{-ax}}{a^4} \\ \int_0^\infty x^{-1/2} e^{-ax} \, dx &= \sqrt{\pi}/a^{1/2} \\ \int_0^\infty x^{1/2} e^{-ax} \, dx &= \sqrt{\pi}/(2a^{3/2}) \\ \int_0^\infty x^n e^{-ax} \, dx &= n!/a^{n+1} \\ \int_0^\infty e^{-ax} \, dx &= 1/a \\ \int_0^\infty x e^{-ax} \, dx &= 1/a^2 \\ \int_0^\infty x^2 e^{-ax} \, dx &= 2/a^3 \\ \int_0^\infty x^3 e^{-ax} \, dx &= 6/a^4 \end{aligned}$$

Trigonometric Functions

$$\begin{aligned} \int \cos x \, dx &= \sin x & \int_0^\pi \cos x \, dx &= 0 \\ \int \sin x \, dx &= -\cos x & \int_0^\pi \sin x \, dx &= 2 \end{aligned}$$