## Chapter 29 Problems: Magnetic Resonance

1. Deuterium NMR, ^{2}H, is quite common. High field instruments have a channel for protons, one or two channels for ^{13}C or other nucleus, and an additional channel for deuterium. The deuterium channel is used for acquiring the resonance frequency of a peak from the deuterated solvent and using that frequency in maintaining a constant magnetic field. This process is called field-locking. The deuterium channel is also used for magnetic field shimming, which adjusts the uniformity of the magnetic field to achieve the narrowest, and hence most intense, transitions. Shimming enhances the resolution of the spectrum. Draw the energy level diagram for deuterium in an applied magnetic field and calculate the resonance frequency of protons and deuterium at a magnetic field of 7.046 T.

Answer: Table 29.1.1 gives the spin quantum number of deuterium as $I = 1$ and the magnetogyric ratio as 41.066×10^6 s^{-1} T^{-1} or equivalently 41.066 MHz T^{-1}. As a result in an applied magnetic field the nuclear energy levels are split into three levels, $m_I = -1, 0, +1$, with equal spacing. The magnetic dipole allowed transitions are for $\Delta m_I = \pm 1$ with the same transition frequency for both $-1 \rightarrow 0$ and $0 \rightarrow 1$ transitions:

$$\nu = \Delta E/h = \gamma_N (2\pi)^{-1} B_o = 41.006 \text{ MHz T}^{-1} (2\pi)^{-1} 7.046 \text{ T} = 45.98 \text{ MHz}$$
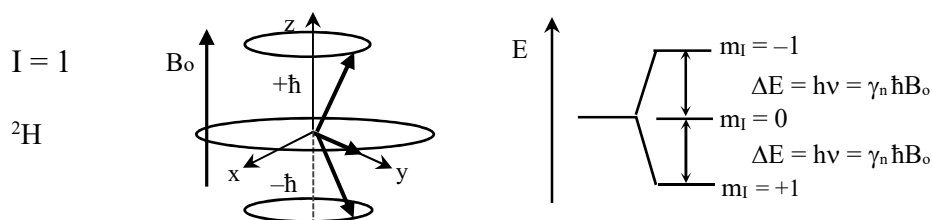
Protons at the same field strength resonate at:

$$\nu = \Delta E/h = \gamma_N (2\pi)^{-1} B_o = 267.522 \text{ MHz T}^{-1} (2\pi)^{-1} 7.046 \text{ T} = 300.0 \text{ MHz}$$

2. The spin quantum number of ^{35}Cl is $I = 3/2$. For a single ^{35}Cl, assuming no spin-spin coupling, how many transitions are observed for ^{35}Cl?

Answer: One transition is observed because the three transitions all have the same frequency. The magnetic dipole allowed transitions are for $\Delta m_I = \pm 1$ giving the three transitions $-3/2 \rightarrow -\frac{1}{2}$, $-\frac{1}{2} \rightarrow +\frac{1}{2}$, and $+\frac{1}{2} \rightarrow +3/2$.

3. A spectral width of at least 10 ppm is required to cover the ^{1}H-NMR spectra of many compounds. (a). Calculate the spectral width in Hz if the resonance frequency of protons is 60 MHz and if the resonance frequency is 500 MHz. (b). The resonance frequency of ^{13}C is ¼ the resonance frequency of protons at the same field strength. A spectral width of 160 ppm is required to cover the ^{13}C-spectra of many compounds. Calculate the spectral width in Hz if the

resonance frequency of ^{13}C is 75 MHz (300 MHz for protons) and if the resonance frequency is 125 MHz (500 MHz for protons).

Answer: The shift in ppm is defined by Eq. 29.1.8: $\delta = [(\nu - \nu_{ref})/\nu_{ref}]$ (1×10^6 ppm). The spectral width corresponds to the maximum plotted value of $\nu - \nu_{ref}$.
(a). At 60 MHz, a 10 ppm spectral width corresponds to 10 ppm(60×10^6 Hz)/1×10^6 ppm =600 Hz and at 500 MHz the spectral width is 5000 Hz.
(b). At 75 MHz, 160 ppm corresponds to 160 ppm(75×10^6 Hz)/1×10^6 ppm = 11,520 Hz and at 125 MHz the spectral width is 20,000 Hz.
   Note that aldehyde protons resonate at 9-10 ppm and carboxylic acid protons resonate in the 10-13 ppm range, so a 10 ppm spectral width is not necessarily sufficient to cover a complete spectrum. We chose a 10 ppm spectral width for this problem to keep the numbers easy to calculate in your head. For ^{13}C spectra carbonyl carbons (C=O) resonate roughly from 150-220 ppm so once again, 160 ppm is not necessarily sufficient to cover a complete spectrum, even though ~160 ppm is a common option for the spectral width in NMR software.

4. In a given instrument, the NMR resonance frequency of a proton in a methyl group is centered at 399,095,832 Hz. The resonance frequency of TMS at the same field strength is 399,095,432 Hz. (a). Calculate the chemical shift of the methyl group. (b). The methyl group is split into a triplet by an adjacent methylene with a spin-spin coupling constant of 7.0 Hz. Calculate the spin-spin splitting in ppm assuming the same resonance frequency as part (a).

Answer: The shift in ppm is defined by Eq. 29.1.8: $\delta = [(\nu - \nu_{ref})/\nu_{ref}]$ (1×10^6 ppm). (a). The chemical shift of the methyl group is:

$$\delta = (399,095,842 - 399,095,432)/399,095,432 \ (1 \times 10^6 \text{ ppm}) = 1.03 \text{ ppm}$$

(b). At a resonance frequency of 400 MHz, a J of 7.0 Hz corresponds to 7.0/400 = 0.0175 ppm

The precise ^1H resonance frequency of a given instrument is essentially never exactly the "name-plate" frequency.

5. The NMR chemical shift of a methyl group is centered at 1.240 ppm at a 60 MHz resonance frequency and the spin-spin splitting constant with an adjacent methylene is 7.0 Hz. (a). Calculate the multiplet peak positions in ppm assuming TMS at 60 MHz. (b). Calculate the multiplet peak positions in ppm with TMS at 500 MHz. (c). Describe the difference in appearance between the spectrum at 60 MHz and 500 MHz.

Answer: An adjacent methylene has two equivalent spins giving the methyl resonance as a triplet. The triplet peak positions are at $\nu_A + J$, ν_A, and $\nu_A - J$ giving the multiplet spacing between the transitions as J.
(a). At 60 MHz, the spin-spin coupling constant of 7.0 Hz corresponds to:

$J = 7.0$ Hz/60×10^6 Hz (1×10^6 ppm) $= 0.117$ ppm

Giving the triplet transitions as: $1.240 + 0.117$ ppm $= 1.357$ ppm, 1.240 ppm, and $1.240 - 0.117$ ppm $= 1.123$ ppm.
(b). At 500 MHz, the spin-spin coupling constant of 7.0 Hz corresponds to:

$J = 7.0$ Hz/500×10^6 Hz (1×10^6 ppm) $= 0.014$ ppm

Giving the triplet transitions as: $1.240 + 0.014$ ppm $= 1.254$ ppm, 1.240 ppm, and $1.240 - 0.014$ ppm $= 1.226$ ppm.
(c). Using the same ppm spectral width, the component transitions of the triplet are closer together at 500 MHz than at 60 MHz, even though the J in Hz is the same, as shown below:

J = 7 Hz at 60 MHz

1.4	1.3	1.2	1.1	1.0 ppm

J = 7 Hz at 500 MHz

1.4	1.3	1.2	1.1	1.0 ppm
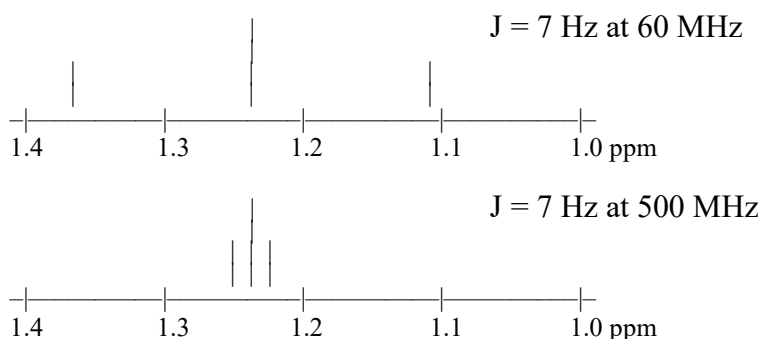
6. (a). Derive Eq. 29.1.7 for the population difference of the spins states of a spin-½ nucleus. (b). Determine the number of spins in the upper and the lower spin states for protons at 400 MHz at 298.2 K. Assume 10^6 total spins. (c). Table-top, permanent magnet NMR spectrometers commonly operate at 60 MHz. Determine the number of spins in the upper and the lower spin states for protons at 60.0 MHz at 298.2 K. Assume 10^6 total spins.

Answer: (a). Given the Boltzmann population ratio, Eq. 29.1.7 and Eq. 29.1.6:

$$\frac{n_-}{n_+} = e^{-\Delta E/kT} = e^{-\gamma \hbar B_\circ/kT} = e^{-V/(kT/h)} \qquad\qquad 1$$

The population difference and sum using Eq. 1 to solve for n_- gives:

$$n_+ - n_- = n_+ - n_+ \,(e^{-V/(kT/h)}) = n_+ \,(1 - e^{-V/(kT/h)})$$

$$n_+ + n_- = n_+ + n_+ \,(e^{-V/(kT/h)}) = n_+ \,(1 + e^{-V/(kT/h)})$$

with $n_{tot} = n_+ + n_-$ as the total number of spins. The ratio is:

$$\frac{n_+ - n_-}{n_+ + n_-} = \frac{1 - e^{-V/(kT/h)}}{1 + e^{-V/(kT/h)}} \qquad \text{giving} \qquad \Delta n = n_+ - n_- = n_{tot}\left(\frac{1 - e^{-V/(kT/h)}}{1 + e^{-V/(kT/h)}}\right)$$

(29.1.7)

The resulting populations are determined using $n_- = n_{tot} - n_+$:

$$\Delta n = n_+ - n_- = n_+ - (n_{tot} - n_+) \qquad \text{or} \qquad n_+ = \tfrac{1}{2}\,(n_{tot} + \Delta n)$$

(b). At 400 MHz and 298.2 K with Eq. 1, the population ratio is $n_-/n_+ = 0.999936$.
Eq. 29.1.7 with $n_{tot} = 10^6$ spins gives the population difference:

$$\Delta n = n_+ - n_- = n_{tot}\left(\frac{1 - 0.999936}{1 + 0.999936}\right) = 32$$

The populations are: $n_+ = \frac{1}{2}(10^6 + 32) = 500016$ and $n_- = 1000000 - 500016 = 499984$.
As a check the population ratio is: $n_-/n_+ = 499984/500016 = 0.999936$ as required.
(c). At the commonly used frequency of 60 MHz, the population ratio is much smaller,
Eq. 29.1.7:

$$\frac{n_-}{n_+} = e^{-V/(kT/h)} = e^{-(60\times10^6\,\text{s}^{-1}/6.2124\times10^{12}\,\text{s}^{-1})} = 0.9999903$$

$$\text{or} \qquad n_+ - n_- = (n_+ + n_-)\left(\frac{1 - 0.9999903}{1 + 0.9999903}\right) = 4.83$$

Out of one million spins there ~5 more spin-ups than spin-downs. No wonder NMR has such
poor sensitivity. At 60 MHz, samples are often run as pure liquids to gain sensitivity. Typical
concentrations in UV/Visible absorption studies are in the 10^{-5} M range. The populations are: $n_+ = \frac{1}{2}(10^6 + 4.83) \cong 500002$ and $n_- = 10^6 - 500002 = 499998$.

7. (a). Give the peak intensities in a sextet that result from coupling to equivalent spins. (b).
Give the peak intensities in a doublet of triplets. (Don't worry about the transition frequencies.)
(c). How many spins are coupled to the observed resonance if the multiplet is a doublet of
quartets? Assume each quartet has intensity ratios: 1:3:3:1.

Answer: (a). Using Pascal's triangle, Table 29.1.5, the intensity ratios of a sextet resulting from
coupling to five equivalent spins are:

$$1 \; : \; 5 \; : \; 10 \; : \; 10 \; : \; 5 \; : \; 1$$

(b). A doublet of triplets has the intensity ratio of 1:2:1—1:2:1, although the transitions may not
arise in that order, see Figure 29.1.10b.
(c). A 1:3:3:1 quartet results from coupling to three equivalent neighbors. A doublet results from
coupling to one near neighbor. In total four spins are coupled to the observed spin. An example
is *trans*-2-butenoic acid (*trans*-crotonic acid). The *trans*-olefinic coupling constant is typically
large: 12-18 Hz:

$$
\begin{array}{ccc}
CH_3 & & H \\
\backslash & & / \\
& C{=}C & \\
/ & & \backslash \\
H & & COOH
\end{array}
$$
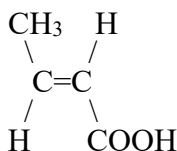
8. Show the spin-spin splitting pattern for nucleus A in the following molecular fragment.
Assume $J_{AB} = 10$ Hz and $J_{AC} = 15$ Hz. Indicate the relative intensities. Assume first-order
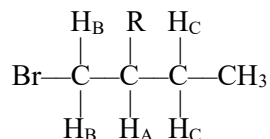behavior.
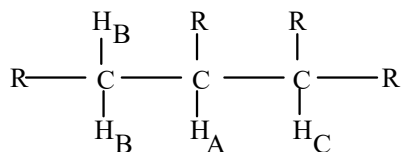
Answer: Construct the coupling tree starting with the largest spin-spin splitting constant. With $J_{AC} = 15$ Hz the resonance is initially split into a doublet with the transitions at +7.5 and –7.5 Hz. With $J_{AB} = 10$ Hz, the previous transitions are split into 1:2:1 triplets: +17.5, 7.5, -2.5, and +2.5, –7.5, –17.5 Hz.

ν: +17.5,		7.5,		-2.5	
and			+2.5,		–7.5,		–17.5 Hz

intensities

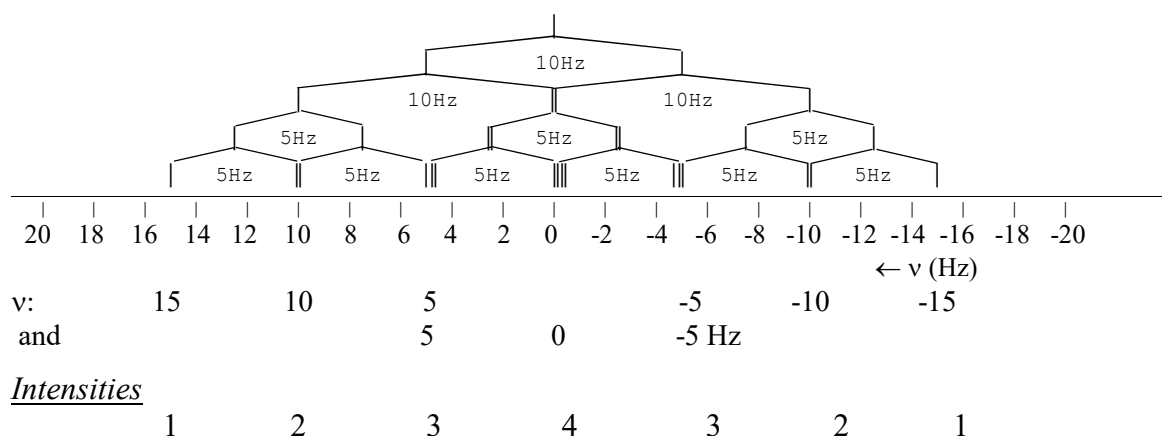| 1	2	1	1	2	1
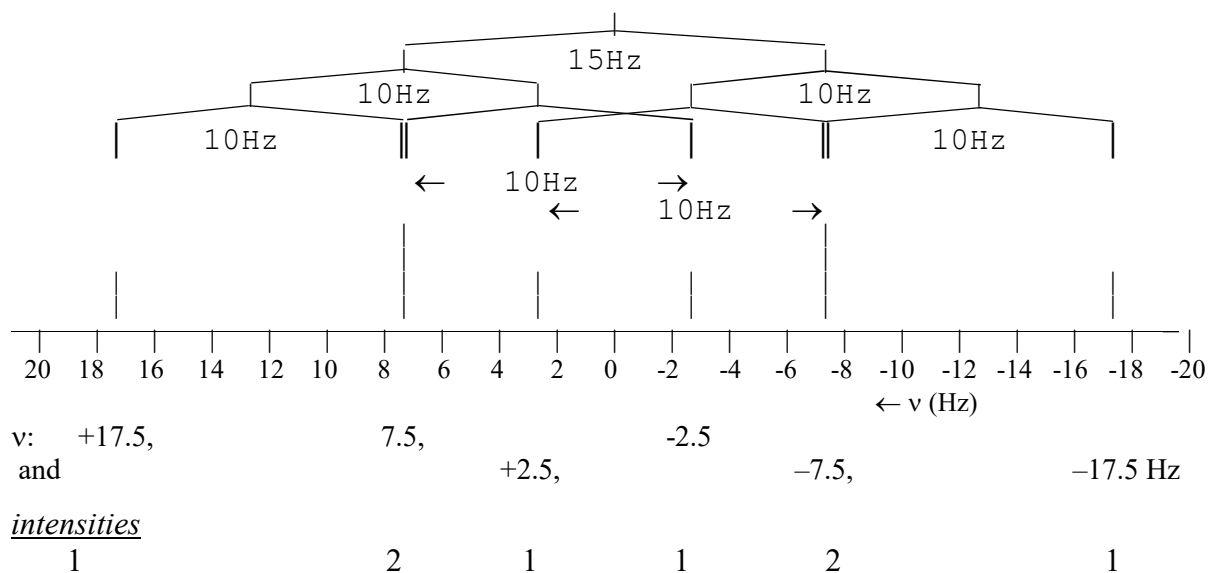
The pattern is a doublet of triplets, or just abbreviated "dt".

9. Show the spin-spin splitting pattern for nucleus A in the following molecular fragment with $J_{AB} = 5$ Hz and $J_{AC} = 10$ Hz. Indicate the relative intensities. Assume first-order behavior.

Answer: Construct the coupling tree starting with the largest spin-spin splitting constant. With $J_{AC} = 10$ Hz the resonance is initially split into a 1:2:1 triplet with the transitions at +10, 0, and –10 Hz. With $J_{AB} = 5$ Hz, the previous transitions are split into a 1:2:1 triplets: +15, 10, 5; 5, 0, -5; and -5, –10, –15 Hz. Because of the coincidences, two pairs of transitions overlap giving a septet with unusual intensities 1:2:3:4:3:2:1. (Note that a septet from six equivalent neighbors gives an intensity ratio of 1:6:15:20:15:6:1, Table 29.1.5.)

10Hz		
10Hz		10Hz
5Hz	5Hz	5Hz
5Hz	5Hz	5Hz	5Hz	5Hz	5Hz

```
|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
20  18  16  14  12  10   8   6   4   2   0  -2  -4  -6  -8 -10 -12 -14 -16 -18 -20
                                                        ← ν (Hz)
```

ν: 15   10   5   -5   -10   -15
and   5   0   -5 Hz

Intensities
_____
                1   2   3   4   3   2   1

The "JMM: First-Order Multiplet Maker" on the course Web site or companion CD (jmmset.html) calculates the first-order multiplet for a given set of neighbors and spin-spin coupling constants. The applet can be used to make suggestions for the values of coupling constants for different bonding environments. The following input, with the specific values of the J-constants entered for this problem, produces the same multiplet as predicted above.

^2J geminal				^3J vicinal			
n	type	phi	J/Hz	n	type	phi	J/Hz
0	alkyl			2	alkyl		10
^4J long range				2	alkyl		5
0	allyl			0	alkyl		
0	allyl			0	alkyl		

10. In an isolated ethyl group, -CH$_2$CH$_3$, there is no spin-spin coupling through the attachment point. Examples include ethyl alcohol, ethylbromide, diethylether, and ethylacetate. An isolated ethyl gives characteristic spin-spin splitting patterns of a quartet and triplet, in the order -CH$_2$CH$_3$. Give the characteristic splitting patterns of isolated *n*-propyl, *iso*-propyl, *n*-butyl, *sec*-butyl, *iso*-butyl, *tert*-butyl, and *iso*-amyl. Assume the vicinal spin-spin coupling constants are approximately equal, with no longer range coupling, and free rotation about the bonds. (Note: *iso*-propyl = 1-methylethyl, *sec*-butyl = 1-methylpropyl, *iso*-butyl = 2-methylpropyl, *tert*-butyl = 1,1-dimethylethane, *iso*-amyl = 3-methylbutyl)

Answer: Assuming all the vicinal, ^3J coupling constants are the same, we use the n+1 rule, counting all the neighbors three bonds away from the given proton. Consider *n*-propyl: the terminal methyl has two near-neighbors giving a triplet, the central methylene has five near-neighbors giving a sextet, and the terminal methylene has two near-neighbors giving a triplet. The following table gives the remaining results.

Isolated Group	Structure	Multiplets	
ethyl	$-CH_2CH_3$	quartet,triplet	
n-propyl	$-CH_2CH_2CH_3$	triplet,sextet,triplet	
iso-propyl	$-CH(CH_3)CH_3$	septet,doublet	(equiv. methyls)
n-butyl	$-CH_2CH_2CH_2CH_3$	triplet,quintet,sextet,triplet	
sec-butyl	$-CH(CH_3)CH_2CH_3$	sextet,quintet,doublet,triplet	
iso-butyl	$-CH_2CH(CH_3)CH_3$	doublet,nonet,doublet	(equiv. methyls)
tert-butyl	$-C(CH_3)(CH_3)CH_3$	singlet	(equiv. methyls)
iso-amyl	$-CH_2CH_2CH(CH_3)CH_3$	triplet,quartet,nonet,doublet	(equiv. methyls)

<u>11</u>. The geminal coupling constant between inequivalent methylene protons on sp³-hybridized carbons is often large, ~12 Hz. (a). Use the Karplus relationship to estimate the J_{AC} and J_{AD} spin-spin coupling constants of the labeled protons in camphor, below. (b). Sketch the expected multiplet pattern of proton-A based on your estimated geminal and vicinal coupling constants. Note that proton-B will also give a similar multiplet that will likely overlap with the multiplet of proton-A. We don't consider the proton-B multiplet in this problem for simplicity. (In practice the appearance of the spectrum is sensitive to the exact values of all the parameters. The purpose of this exercise is to give just one reasonable prediction of the appearance of the spectrum.)
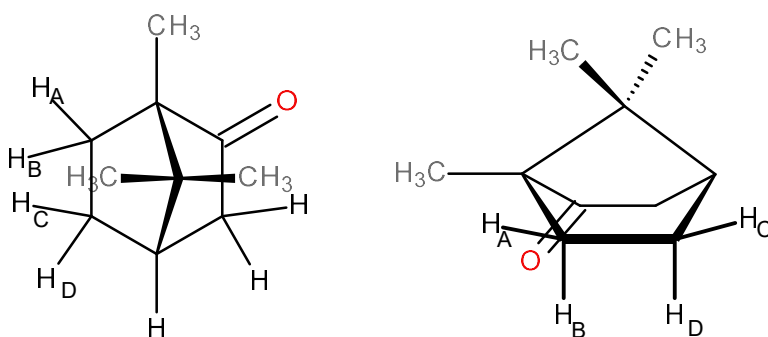

Dihedral angles: $\phi_{AC} = 0°$ and $\phi_{AC} = 120°$

Answer: (a). The Karplus relationship is given by Eqs. 29.1.16:

$$J_{HH} = 8.5 \cos^2\phi - 0.28 \qquad\qquad 0° \le \phi \le 90°$$
$$J_{HH} = 9.5 \cos^2\phi - 0.28 \qquad\qquad 90° \le \phi \le 180°$$

For $\phi_{AC} = 0°$: $J_{AC} = 8.22$ Hz and $\phi_{AD} = 120°$: $J_{AD} = 2.095$ Hz.

(b). Given that J_{AB} , J_{AC}, and J_{AD} are all different, the spectrum is predicted to have $2^3 = 8$ transitions all of equal intensity (no overlapping of transitions). Start the coupling tree with the largest coupling constant. With $J_{AB} = 12$ Hz the resonance is initially split into a doublet with the transitions at +6 and –6 Hz. With $J_{AC} = 8.22$ Hz, the previous transitions are split into doublets: 10.11, 1.89; and -1.80, -10.11 Hz. With $J_{AD} = 2.095$ Hz, the final doublets are: 11,16, 9.06; 2.94, 0.84; -0.84, -2.94; and -9.06, -11.16 Hz.
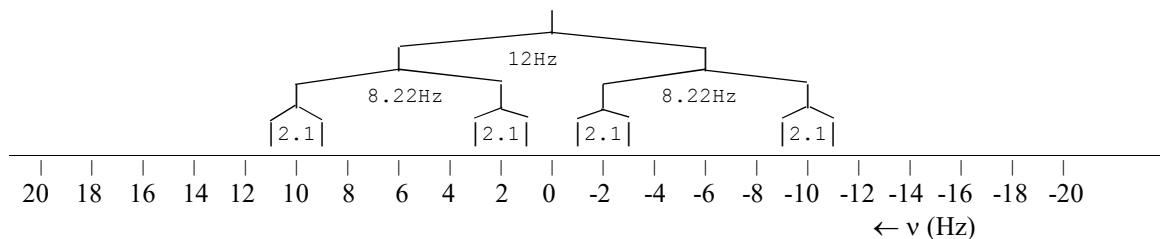
Another commonly quoted form of the Karplus relationship is:

$$^3J = 7.8 - 1.0 \cos \phi + 5.6 \cos 2\phi$$

This form is implemented in the "*Vicinal Spin-Spin Coupling Constant Prediction*" applet (altona.html) on the textbook Web site or companion CD. The predicted values based on this version of the Karplus equation are $\phi_{AC} = 0°$: $J_{HH} = 8.06$ Hz and $\phi_{AD} = 120°$: 3.89 Hz. This applet also implements several more accurate prediction equations that allow for the electron withdrawing effects of neighboring substituents. You should use molecular mechanics to verify the dihedral angles that were given in this problem.

The "*JMM: First-Order Multiplet Maker*" on the course Web site or companion CD (jmmset.html) calculates the first-order multiplet for a given set of neighbors and spin-spin coupling constants. The applet can additionally be used to make suggestions for the values of coupling constants of different bonding environments including dihedral angles. The following input, with the specific values of the J-constants entered for this problem, produces the same multiplet as predicted above.

^2J geminal				^3J vicinal			
n	type	phi	J/Hz	n	type	phi	J/Hz
1	alkyl			1	alkyl		8.22
^4J long range				1	alkyl		2.95
0	allyl			0	alkyl		
0	allyl			0	alkyl		

12. Determine the structure of the following compound. The spectrum was acquired at 270 MHz. The degree of unsaturation is a useful starting point if the formula of the compound is known. The degree of unsaturation is equal to the sum of the number of double bonds and rings: $dbr = (2c - h + 2 + n - x)/2$, where c is the number of carbons, h the number of hydrogens, n the number of nitrogens, and x the number of monovalent atoms, which includes F, Cl, and I.
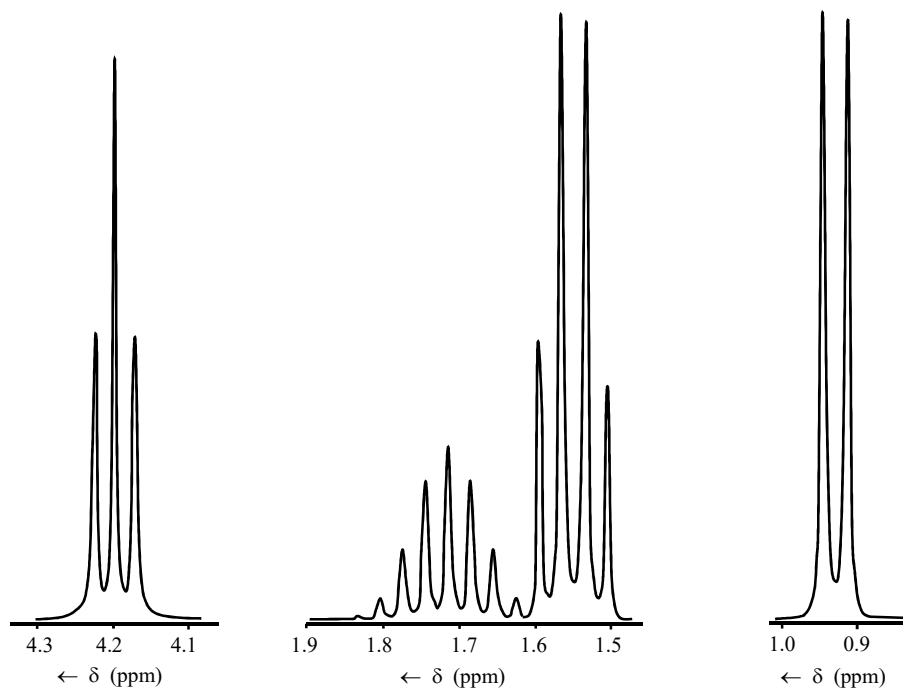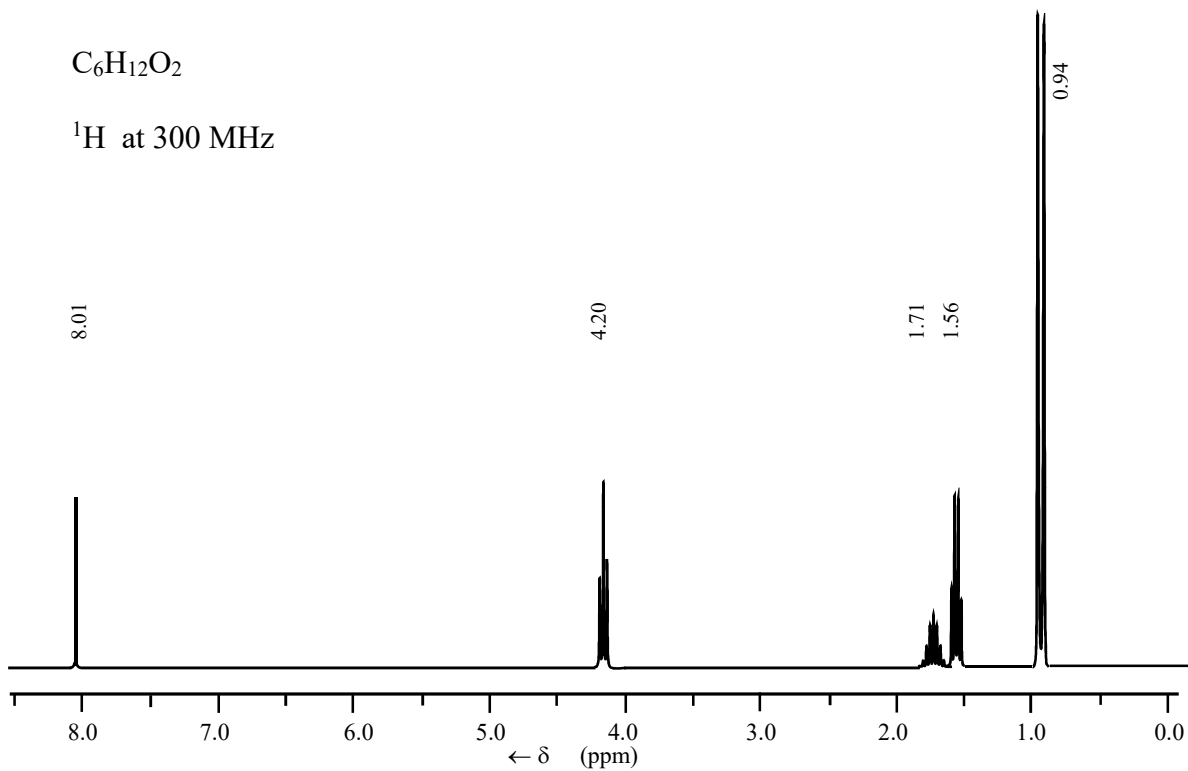
C$_6$H$_{12}$O$_2$

^{1}H at 270 MHz
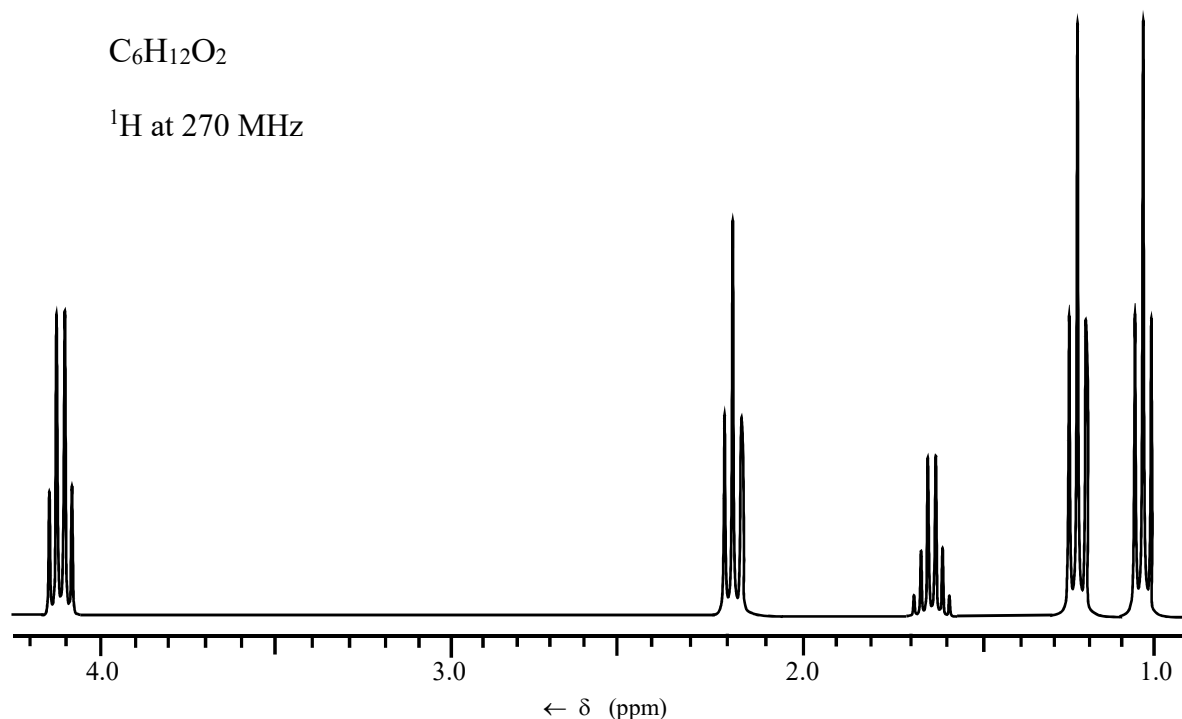
Answer: The number of double bonds and rings is: dbr = $(2 \cdot 6 - 12 + 2 + 0 - 0)/2 = 1$. An alkene, aldehyde, carboxylic acid, or ester are possible functional groups or a single saturated ring. The typical approach is to begin with the largest chemical shifts. The triplet at 4.1 ppm requires interaction with a nearby heteroatom or that the proton is on a C=C, Table 29.1.3. The chemical shift agrees with the 4.1 ppm entry on Table 29.1.2 for a methylene that is the –O– linked portion of an ester: "R-CH$_2$-O-C(=O)-R". The appearance of a quartet and a triplet in a spectrum is often a strong indication of an ethyl group. The triplet at 1.23 ppm agrees with the 1.3 ppm entry on Table 29.1.2 for a methyl that is beta to an –O–: "CH$_3$–C-O-". As a result we suspect that the compound is an ethyl ester. The triplet near 2.2 ppm agrees with the 2.2 ppm entry on Table 29.1.2 for a methylene that is the –C(=O)–OR linked portion of an ester: "R–CH$_2$-CO–OR". The sextet at 1.63 ppm requires coupling to five similar neighbors, which in this case results from CH$_3$CH$_2$CH$_2$–. The terminal position of this three-carbon substituent produces the remaining triplet. The spectrum is then consistent with ethyl butyrate: CH$_3$CH$_2$CH$_2$C(=O)OCH$_2$CH$_3$.
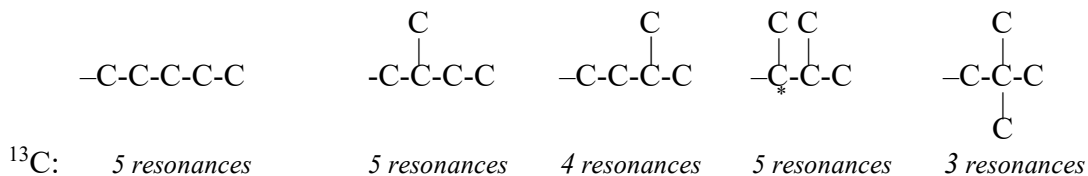
  Note that an aldehyde or carboxylic acid would have a resonance at 9 ppm or below. Protons attached to a double bond are typically down field of 4.1 ppm, Table 29.1.3, although "–HC=C–O–" is a possibility to consider in this example.

<u>13</u>. Determine the structure of the following compound. The spectrum was acquired at 300 MHz. The down field resonance at 8.01 ppm is a singlet. Expanded spectra of the multiplets are shown below the full spectrum. The ^{13}C spectrum has five peaks with the most downfield peak at 161.2 ppm. [Hint: see the comments about the degree of unsaturation in the previous problem.]

$C_6H_{12}O_2$

^1H at 300 MHz

8.01

4.20

1.71
1.56

0.94

8.0   7.0   6.0   5.0   4.0   3.0   2.0   1.0   0.0

← δ   (ppm)

4.3   4.2   4.1

← δ (ppm)

1.9   1.8   1.7   1.6   1.5

← δ (ppm)

1.0   0.9

← δ (ppm)

Answer: The ^{13}C spectrum has five peaks, which shows that the compound has two magnetically equivalent carbons. The proton spectrum also has five resonances, which again shows that there are two magnetically equivalent groups. As in the previous problem, the number of double bonds

and rings, is dbr $= (2 \cdot 6 - 12 + 2 + 0 - 0)/2 = 1$. An alkene, aldehyde, carboxylic acid, or ester are possible functional groups or a single saturated ring. The typical approach is to begin with the largest chemical shifts. The proton singlet at 8.01 ppm is consistent with a formate proton, which are expected in the range of 8.0-8.2, Table 29.1.3. Similarly, the down field ^{13}C shift of 161.2 ppm matches the shift of ethyl formate at 161.4 ppm, Table 29.1.4. We now must account for five carbons, two of which are equivalent, with 11 attached protons. Of the possible five-carbon fragments:

$$
\begin{array}{ccccc}
 & C & C & C\ C & C \\
 & | & | & |\ | & | \\
-C\text{-}C\text{-}C\text{-}C\text{-}C & -C\text{-}C\text{-}C\text{-}C & -C\text{-}C\text{-}C\text{-}C & -C\text{-}C\text{-}C & -C\text{-}C\text{-}C \\
 & & & {}^{*} & | \\
 & & & & C
\end{array}
$$

^{13}C: *5 resonances*   *5 resonances*   *4 resonances*   *5 resonances*   *3 resonances*

only the third possibility, the *iso*-pentyl group, has two and only two equivalent carbons. The 2,3-dimethylpropyl fragment has a chiral center, *, so the terminal –CH$_3$ groups are inequivalent. The proton spin-spin multiplicities expected for the *iso*-pentyl group are indicated below and are also consistent with the spectrum:

$$
\begin{array}{c}
CH_3 \\
| \\
-CH_2\text{-}CH_2\text{-}CH\text{-}CH_3
\end{array}
$$

multiplicity: (3) (4) (9) (2)
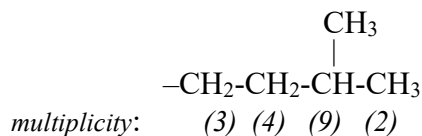
The outer peaks of a nonet are often difficult to see, as they are often obscured by noise. The intensity ratio is unfavorable for detecting the outer transitions: 1:8:28:56:70:56:28:8:1. In this case the ninth transition overlaps with an outer peak of the quartet. However, given that a multiplet from n-equivalent neighbors is always symmetrical, you can determine the multiplicity just from the left side of the multiplet. From Table 29.1.2 the expected chemical shift of the methylene protons adjacent to the HC(=O)-O– of the formate group is 4.1 ppm, as listed for "R–CH$_2$–O–C(=O)–R". The tabulated value is sufficiently close to the observed 4.2 ppm to have confidence in the assignment. The compound is *iso*-pentyl formate:

$$
\begin{array}{cc}
O & CH_3 \\
\| & | \\
H\text{-}C\text{-}O\text{-}CH_2\text{-}CH_2\text{-}CH\text{-}CH_3
\end{array}
$$
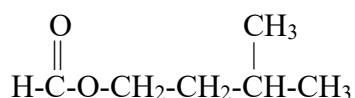
<u>14</u>. Determine the number of proton and carbon resonances that are observed for the following compounds. Determine the corresponding multiplicities of the spin-spin coupling multiplets.
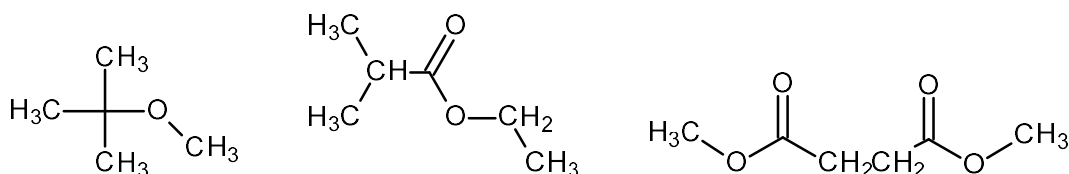
Answer: For methyl-*tert*-butylether the three *tert*-butyl groups are magnetically equivalent giving two proton chemical shifts and three ^{13}C chemical shifts. The two proton environments result in singlets.

For ethyl *iso*-butyrate the two methyls in the *iso*-butyrate side of the molecule are magnetically equivalent giving four proton chemical shifts and five ^{13}C chemical shifts (don't forget the carbonyl carbon). The ethyl group gives a quartet and triplet. An isolated *iso*-butyrate group gives a doublet and septet.

Dimethyl succinate has a mirror plane giving two proton chemical shifts and three ^{13}C chemical shifts. Both proton resonances are singlets (magnetically equivalent protons don't split).

<u>15</u>. *Equivalent Spins Don't Split*: Derive the energy levels and transition frequencies of a system with two equivalent protons. This problem fills in the details of the energy level diagram in Figure 29.1.13. Because the chemical shift differences are not greater than the spin-spin coupling constant, the full spin-spin coupling Hamiltonian must be used for this problem:

$$\hat{\mathcal{H}} = -\nu_A \hat{I}_{zA} - \nu_B \hat{I}_{zB} + J_{AB} \vec{I}_A \cdot \vec{I}_B \qquad (29.1.13)$$

where $\vec{I}_A \cdot \vec{I}_B = \hat{I}_{xA} \hat{I}_{xB} + \hat{I}_{yA} \hat{I}_{yB} + \hat{I}_{zA} \hat{I}_{zB}$

The allowable spin states for two equivalent spins are the symmetric combinations: $\alpha\alpha$, $1/\sqrt{2}\,(\alpha\beta + \beta\alpha)$, and $\beta\beta$, while the fourth spin state is antisymmetric: $1/\sqrt{2}\,(\alpha\beta - \beta\alpha)$. In Ch. 24.7 we found the relationship between the angular momentum raising and lowering operators and the x and y-components of the angular momentum, Eq. 24.7.18. Expressed explicitly in terms of nuclear angular momentum operators, Eqs. 24.7.18 are recast as:

$$\hat{I}^- = \hat{I}_x - i\,\hat{I}_y \quad \text{(lowering)} \qquad \text{and} \qquad \hat{I}^+ = \hat{I}_x + i\,\hat{I}_y \quad \text{(raising)} \qquad (24.7.18)$$

which have the following effects on the spin wave functions:

$$\hat{I}^+ \alpha = 0 \qquad\qquad \hat{I}^+ \beta = \alpha$$
$$\hat{I}^- \alpha = \beta \qquad\qquad \hat{I}^+ \beta = 0$$

Solving for $\hat{I}_x$ and $\hat{I}_y$ gives for both A and B spins:

$$\hat{I}_x = \tfrac{1}{2}(\hat{I}^+ + \hat{I}^-) \qquad\qquad \text{and} \qquad \hat{I}_y = \tfrac{1}{2}\,i(\hat{I}^+ - \hat{I}^-)$$

In a subsequent problem we will prove that the spin-spin interaction can be written as:

$$J_{AB} \vec{I}_A \cdot \vec{I}_B = J_{AB} [\,\hat{I}_{zA} \hat{I}_{zB} + \tfrac{1}{2}(\hat{I}_A^+ \hat{I}_B^- + \hat{I}_A^- \hat{I}_B^+)]$$

(a). For simplicity of notation set $J = J_{AB}$. Show that:

$$J \vec{I}_A \cdot \vec{I}_B\; \alpha\alpha = \; {}^J/_4\; \alpha\alpha$$
$$J \vec{I}_A \cdot \vec{I}_B\; \alpha\beta = -{}^J/_4\; \alpha\beta + {}^J/_2\; \beta\alpha$$
$$J \vec{I}_A \cdot \vec{I}_B\; \beta\alpha = -{}^J/_4\; \beta\alpha + {}^J/_2\; \alpha\beta$$
$$J \vec{I}_A \cdot \vec{I}_B\; \beta\beta = \; {}^J/_4\; \beta\beta$$

Answer: The eigenfunctions are the spin-A spin-B products: $\alpha\alpha$, $1/\sqrt{2}(\alpha\beta+\beta\alpha)$, $1/\sqrt{2}(\alpha\beta-\beta\alpha)$, $\beta\beta$. $\hat{I}_A^+$ and $\hat{I}_A^-$ only operate on the coordinates of spin-A and $\hat{I}_B^+$ and $\hat{I}_B^-$ only operate on the

coordinates of spin-B. For example, using for a moment explicit A and B designations, the action of the raising and lowering operators on $\alpha\beta = \alpha_A\beta_B$ are:

$$\hat{I}_A^+ \hat{I}_B^- \alpha_A\beta_B = (\hat{I}_A^+ \alpha_A)(\hat{I}_B^- \beta_B) = 0 \cdot 0 \qquad\qquad \hat{I}_A^- \hat{I}_B^+ \alpha_A\beta_B = (\hat{I}_A^- \alpha_A)(\hat{I}_B^+ \beta_B) = \beta_A\alpha_B$$

The results of the raising and lowering operators on the products are:

Ψ	$\hat{I}_A^+ \hat{I}_B^-$	$\hat{I}_A^- \hat{I}_B^+$
$\alpha\alpha$	0	0
$\alpha\beta$	0	$\beta\alpha$
$\beta\alpha$	$\alpha\beta$	0
$\beta\beta$	0	0
$1/\sqrt{2}(\alpha\beta+\beta\alpha)$	$1/\sqrt{2}\,\alpha\beta$	$1/\sqrt{2}\,\alpha\beta$
$1/\sqrt{2}(\alpha\beta-\beta\alpha)$	$1/\sqrt{2}\,\alpha\beta$	$-1/\sqrt{2}\,\alpha\beta$

Expanding the terms in the spin-spin interaction, $J_{AB}[\,\hat{I}_{zA}\hat{I}_{zB} + \tfrac{1}{2}(\hat{I}_A^+ \hat{I}_B^- + \hat{I}_A^- \hat{I}_B^+)]$, gives:

$$J\,\vec{I}_A\cdot\vec{I}_B\,\alpha\alpha = J\,\hat{I}_{zA}\hat{I}_{zB}\,\alpha\alpha + \tfrac{1}{2}J\,\hat{I}_A^+\hat{I}_B^-\,\alpha\alpha + \tfrac{1}{2}J\,\hat{I}_A^-\hat{I}_B^+\,\alpha\alpha = \; {}^J/_4\,\alpha\alpha$$

$$J\,\vec{I}_A\cdot\vec{I}_B\,\alpha\beta = J\,\hat{I}_{zA}\hat{I}_{zB}\,\alpha\beta + \tfrac{1}{2}J\,\hat{I}_A^+\hat{I}_B^-\,\alpha\beta + \tfrac{1}{2}J\,\hat{I}_A^-\hat{I}_B^+\,\alpha\beta = -{}^J/_4\,\alpha\beta + {}^J/_2\,\beta\alpha$$

$$J\,\vec{I}_A\cdot\vec{I}_B\,\beta\alpha = J\,\hat{I}_{zA}\hat{I}_{zB}\,\beta\alpha + \tfrac{1}{2}J\,\hat{I}_A^+\hat{I}_B^-\,\beta\alpha + \tfrac{1}{2}J\,\hat{I}_A^-\hat{I}_B^+\,\beta\alpha = -{}^J/_4\,\beta\alpha + {}^J/_2\,\alpha\beta$$

$$J\,\vec{I}_A\cdot\vec{I}_B\,\beta\beta = J\,\hat{I}_{zA}\hat{I}_{zB}\,\beta\beta + \tfrac{1}{2}J\,\hat{I}_A^+\hat{I}_B^-\,\beta\beta + \tfrac{1}{2}J\,\hat{I}_A^-\hat{I}_B^+\,\beta\beta = \; {}^J/_4\,\beta\beta$$

(b). Combine these results to show that:

$$J\,\vec{I}_A\cdot\vec{I}_B\,[1/\sqrt{2}(\alpha\beta+\beta\alpha)] = \; {}^J/_4\,[1/\sqrt{2}\,(\alpha\beta + \beta\alpha)]$$

$$J\,\vec{I}_A\cdot\vec{I}_B\,[1/\sqrt{2}(\alpha\beta-\beta\alpha)] = -3^J/_4\,[1/\sqrt{2}\,(\alpha\beta - \beta\alpha)]$$

Answer: Combining the previous results gives:

$$J\,\vec{I}_A\cdot\vec{I}_B\,[1/\sqrt{2}(\alpha\beta+\beta\alpha)] = 1/\sqrt{2}\,(-{}^J/_4\,\alpha\beta + {}^J/_2\,\beta\alpha - {}^J/_4\,\beta\alpha + {}^J/_2\,\alpha\beta) = {}^J/_4\,[1/\sqrt{2}\,(\alpha\beta + \beta\alpha)]$$

$$J\,\vec{I}_A\cdot\vec{I}_B\,[1/\sqrt{2}(\alpha\beta-\beta\alpha)] = 1/\sqrt{2}\,(-{}^J/_4\,\alpha\beta + {}^J/_2\,\beta\alpha + {}^J/_4\,\beta\alpha - {}^J/_2\,\alpha\beta) = -3^J/_4\,[1/\sqrt{2}\,(\alpha\beta - \beta\alpha)]$$

(c). Use these results to verify the energies of the levels shown in Figure 29.1.13.

Answer: Using the results of part (b), the symmetric and antisymmetric combinations are seen to be eigenfunctions of the spin-spin coupling operator. In other words, the same wave function appears on both sides of each expression. The eigenvalues are $+{}^J/_4$ for the symmetric combination and $-3^J/_4$ for the antisymmetric combination, verifying Figure 29.1.13.

<u>16</u>. Using the relationships given in the introduction to the last problem to prove that the spin-spin interaction can be written as:

$$J_{AB} \vec{I}_A \cdot \vec{I}_B = J_{AB} [\hat{I}_{zA} \hat{I}_{zB} + \tfrac{1}{2}(\hat{I}_A^+ \hat{I}_B^- + \hat{I}_A^- \hat{I}_B^+)]$$

Answer: Expanding the dot product into separate components:

$$J_{AB} \vec{I}_A \cdot \vec{I}_B = J_{AB} (\hat{I}_{xA} \hat{I}_{xB} + \hat{I}_{yA} \hat{I}_{yB} + \hat{I}_{zA} \hat{I}_{zB}) \qquad\qquad 1$$

The z-component remains unchanged. However we note from the previous problem that $\hat{I}_x = \tfrac{1}{2}(\hat{I}^+ + \hat{I}^-)$ and $\hat{I}_y = \tfrac{1}{2} i(\hat{I}^+ - \hat{I}^-)$ and:

$$
\begin{aligned}
\hat{I}_{xA} \hat{I}_{xB} + \hat{I}_{yA} \hat{I}_{yB} &= \tfrac{1}{2}(\hat{I}_A^+ + \hat{I}_A^-)\tfrac{1}{2}(\hat{I}_B^+ + \hat{I}_B^-) + \tfrac{1}{2} i(\hat{I}_A^+ - \hat{I}_A^-)\tfrac{1}{2} i(\hat{I}_B^+ - \hat{I}_B^-) \\
&= \tfrac{1}{4}(\hat{I}_A^+ + \hat{I}_A^-) (\hat{I}_B^+ + \hat{I}_B^-) - \tfrac{1}{4} (\hat{I}_A^+ - \hat{I}_A^-) (\hat{I}_B^+ - \hat{I}_B^-) \\
&= \tfrac{1}{4}(\hat{I}_A^+\hat{I}_B^+ + \hat{I}_A^+\hat{I}_B^- + \hat{I}_A^-\hat{I}_B^+ + \hat{I}_A^-\hat{I}_B^-) - \tfrac{1}{4} (\hat{I}_A^+\hat{I}_B^+ - \hat{I}_A^+\hat{I}_B^- - \hat{I}_A^-\hat{I}_B^+ + \hat{I}_A^-\hat{I}_B^-) \qquad 2
\end{aligned}
$$

Cancelling terms gives:

$$\hat{I}_{xA} \hat{I}_{xB} + \hat{I}_{yA} \hat{I}_{yB} = \tfrac{1}{4}(\hat{I}_A^+\hat{I}_B^- + \hat{I}_A^-\hat{I}_B^+) - \tfrac{1}{4} (- \hat{I}_A^+\hat{I}_B^- - \hat{I}_A^-\hat{I}_B^+) = \tfrac{1}{2}(\hat{I}_A^+ \hat{I}_B^- + \hat{I}_A^- \hat{I}_B^+) \qquad 3$$

Substituting Eq. 3 into Eq. 1 gives the final result: $J_{AB} \vec{I}_A \cdot \vec{I}_B = J_{AB} [\hat{I}_{zA} \hat{I}_{zB} + \tfrac{1}{2}(\hat{I}_A^+ \hat{I}_B^- + \hat{I}_A^- \hat{I}_B^+)]$

<u>17</u>. Consider the spin-spin coupling of two inequivalent spins. If the difference in chemical shift is much larger than the spin-spin coupling constant then Eq. 29.1.14 is a good approximation. As seen in Figure 29.1.11, the wave functions αα, αβ, βα, and ββ are then good eigenfunctions of the approximate Hamiltonian, Eq. 29.1.14. If the chemical shift difference is comparable to the spin-spin coupling constant, then the exact Hamiltonian must be used, Eq. 29.1.13. Use the results of Problem 15(b) to determine if αα, αβ, βα, and ββ are eigenfunctions of the exact Hamiltonian. If αα, αβ, βα, and ββ are eigenfunctions of the exact Hamiltonian, then they may be used to determine the energies of the final spin levels directly from the eigenvalues, as we did in Figure 29.1.11. If αα, αβ, βα, and ββ are not eigenfunctions of the exact Hamiltonian, then the exact wave functions will be linear combinations of αα, αβ, βα, and ββ. The wave functions must be determined from the secular equation that is based on the exact Hamiltonian.

Answer: The wave functions αα, αβ, βα, and ββ are eigenfunctions of the portion of the Hamiltonian that describes the interaction of the spins with the external magnetic field, which is called the Zeeman interaction:

$$\hat{\mathcal{H}}(Zeeman) = - \nu_A \hat{I}_{zA} - \nu_B \hat{I}_{zB}$$

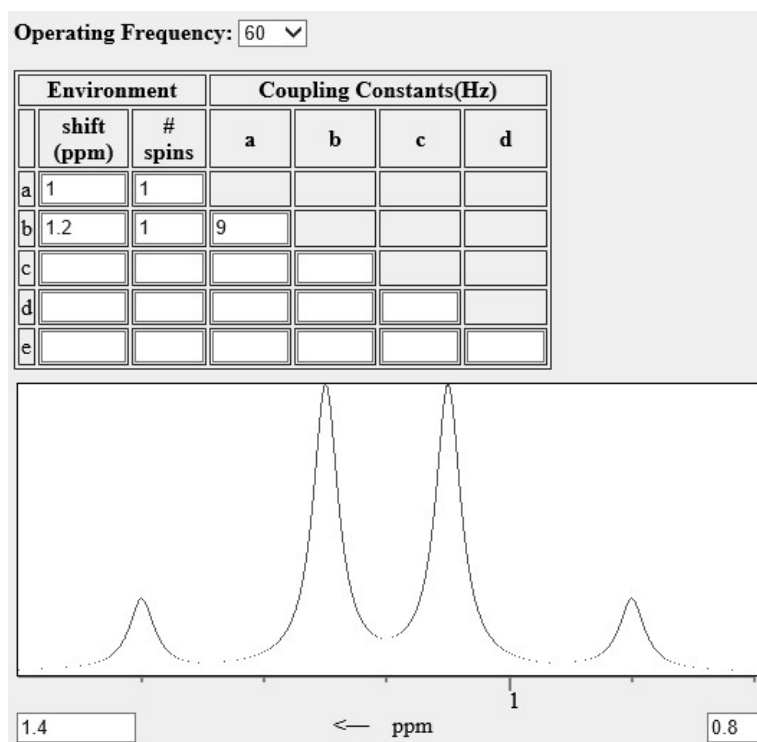However, the results of Problem 15(b) show that αβ and βα are not eigenfunctions of the exact spin-spin Hamiltonian:

$$J \vec{I}_A \cdot \vec{I}_B \, \alpha\beta = - {}^J\!/_4 \, \alpha\beta + {}^J\!/_2 \, \beta\alpha$$
$$J \vec{I}_A \cdot \vec{I}_B \, \beta\alpha = - {}^J\!/_4 \, \beta\alpha + {}^J\!/_2 \, \alpha\beta$$

In other words just αβ alone does not appear on each side of the first expression and just βα alone does not appear on each side of the second expression. As a result, αβ and βα mix in linear

combination to form the eigenfunctions of the exact Hamiltonian in the final results. Computer applications are available to do these exact calculations. The "*JD: Spin-Spin Splitting Simulation*" applet on the course Web site or companion CD is available to do these calculations.

18. Use the "*JD: Spin-Spin Splitting Simulation*" applet (jdplot.html) on the course Web site or companion CD to determine the spectrum of an ^{1}H AB-system. An AB-system is comprised of two inequivalent spin-spin coupled protons, with the difference in chemical shifts between the two protons comparable to the spin-spin coupling constant. Assume the two chemical shifts are 1.00 ppm and 1.20 ppm, with the resonance frequency at 60.000 MHz. Use a spin-spin coupling constant of J = 9.0 Hz. Compare the results to the first-order predictions based on the energy levels derived in Figure 29.1.11 and 29.1.12.

Answer: The input and output from the jdplot.html applet are shown below. The results are summarized in the table, below.

Operating Frequency: 60

	Environment		Coupling Constants(Hz)			
	shift (ppm)	# spins	a	b	c	d
a	1	1				
b	1.2	1	9			
c						
d						
e						

1.4   ⟵ ppm   0.8

```
Transitions (ppm) and Intensities:
  1.05  :     1.6
  1.3   :     0.4
  1.15  :     1.6
  0.9   :     0.4
```

   The first-order prediction corresponds to two doublets, the first centered at 1.00 ppm and the second centered at 1.2 ppm. The spin-spin coupling constant of 9.00 Hz at a resonance frequency of 60.000 MHz corresponds to $9.00/60.000 \times 10^6$ (1×10^6 ppm) = 0.15 ppm. The first-order

transitions are at $\nu_A \pm J_{AB}/2$ and $\nu_B \pm J_{AB}/2$, as given in the table, below. The first-order transition intensities are all the same, 1:1:1:1.
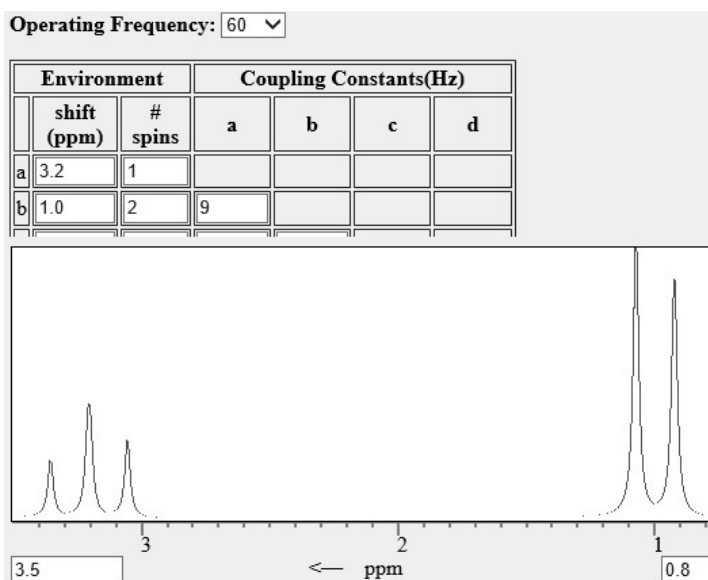
Proton	First Order	1st order (ppm)	Exact (ppm)	Exact – 1storder	Intensity
A	$\nu_A + J_{AB}/2$	1.275	1.3	0.025	0.4
	$\nu_A - J_{AB}/2$	1.125	1.15	0.025	1.6
B	$\nu_A + J_{AB}/2$	1.075	1.05	-0.025	1.6
	$\nu_A - J_{AB}/2$	0.925	0.9	-0.25	0.4

The differences between the exact spin-spin splitting pattern and the first-order prediction are called second-order effects. One second-order effect for an AB-spectrum is that the doublets are no longer centered on the chemical shift values. This effect is shown by the differences shown in the table with the heading: Exact – 1storder. The A-doublet transitions move down-field relative to the chemical shift, which is 1.20 ppm. The B-doublet transitions move up-field relative to the chemical shift, which is 1.00 ppm. The other second-order effect is that the "inner" transitions are higher in intensity than the "outer" transitions. The stronger the second-order effect the larger the difference in intensity of the inner and outer transitions in each doublet. This shift in intensity is commonly observed with multiplets of all kinds, not just doublets.

19. A surprising result of strong second-order effects in spin-spin splitting is that more transitions appear than expected based on first-order analysis. Consider a ^{1}H AB$_2$ pattern as an example. An AB$_2$ pattern corresponds to two chemical environments, A with one proton and B with two protons, with the difference in chemical shifts between the two environments comparable to the spin-spin coupling constant. In comparison, an AX$_2$ pattern corresponds to the same proton distribution but with the difference in chemical shifts between the two environments much larger than the spin-spin coupling constant. (a). Use the "*JD: Spin-Spin Splitting Simulation*" applet (jdplot.html) on the course Web site or companion CD to determine the spectrum of an ^{1}H AX$_2$-system. Assume the two chemical shifts are 3.20 ppm and 1.00 ppm, with the resonance frequency at 60.000 MHz. Use a spin-spin coupling constant of J = 9.0 Hz. Does the resulting spectrum agree with the first-order prediction? (b). Determine the spectrum of an ^{1}H AB$_2$-system. Assume the two chemical shifts are 1.20 ppm and 1.00 ppm, with the resonance frequency at 60.000 MHz. Once again, use a spin-spin coupling constant of J = 9.0 Hz. Decrease the line width to 0.75 Hz to better observe the number of transitions. How many transitions are evident?
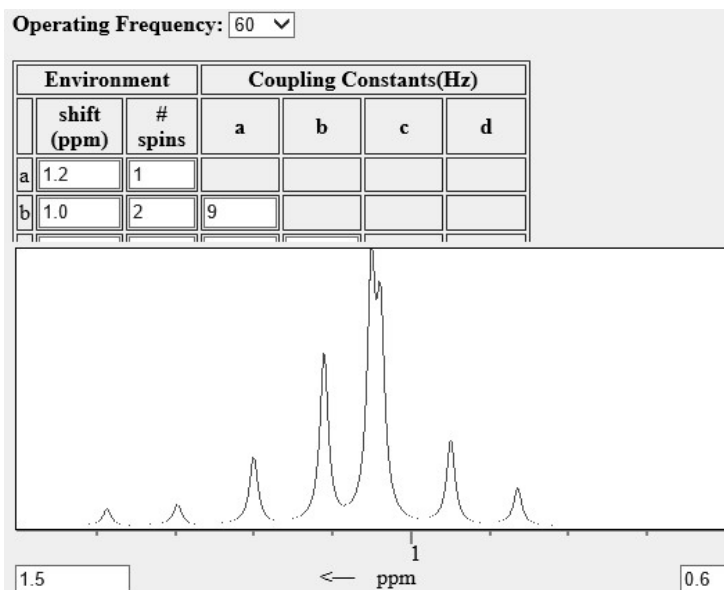
Answer: (a). The input and output from the jdplot.html applet for the AX$_2$ case are shown below.

Operating Frequency: 60 ▾

Environment		Coupling Constants(Hz)			
shift (ppm)	# spins	a	b	c	d
a 3.2	1				
b 1.0	2	9			

3.5	← ppm	0.8

The first-order prediction is an A-triplet and B-doublet, based on the n + 1 near-neighbor rule. This expectation is observed giving a total of <u>five</u> observed transitions. [Replotting the spectrum at 0.25 Hz line width shows that the middle two peaks in the triplet don't quite overlap, giving six transitions at very high resolution. However, a resolution of 0.25 Hz is hard to achieve because 0.25 Hz resolution requires very good magnet homogeneity.]

(b). The input and output from the jdplot.html applet for the AB_2 case are shown below.

Operating Frequency: 60 ▾

Environment		Coupling Constants(Hz)			
shift (ppm)	# spins	a	b	c	d
a 1.2	1				
b 1.0	2	9			

1.5	← ppm	0.6

For the AB_2 case <u>eight</u> transitions are apparent. The numerical listing has nine transitions, however. The transition at 0.703 ppm has low intensity. This weak transition is likely to be lost in the baseline noise in practical spectra.

  The spectra of aromatic compounds is often complicated by second-order effects, because the chemical shift differences among aromatic chemical environments is often small and the spin-spin coupling constants can be large. Second-order effects are lessened by high-field instruments.

20. In pulsed NMR, the free induction decay of all the chemical shifts in the spectrum are excited by a short pulse at a single frequency. For example, the proton chemical shift range is ~5000 Hz at a resonance frequency of 400.00 MHz, while the pulse is at a single frequency of 400.000 MHz. Explain how all the chemical shifts in the spectrum can be excited by a short pulse at a single frequency. Assume the pulse length is 15.0 μs.

Answer: The Fourier transform of a short pulse has a wide range of component frequencies, even though the pulse itself is constructed by chopping a sine or cosine wave of a single central frequency. In other words, to reconstruct the rectangular pulse, the superposition of a wide range of frequencies is necessary to reproduce the time dependence of the pulse. The range of frequencies is approximately characterized by the width of the pulse in Hz to the first nulls of the Fourier transform, Figure 27.3.3, $\Delta\nu = 1/t_p$ where t_p is the duration of the rectangular pulse. If the pulse length is 15.0 μs, the frequency range to the first nulls is $1/15.0 \times 10^{-6}$ s $= 6.67 \times 10^4$ Hz, which is roughly a factor of ten larger than the expected ^{1}H-chemical shift range.

Note that the chemical shift range of ^{13}C is roughly 20-fold larger than the range of ^{1}H shifts. As a result, care must be taken in ^{13}C experiments to maintain short pulse widths. To minimize the effects of the bandwidth of the excitation pulse, the frequency of the NMR transmitter is usually set to the middle of the chemical shift range, rather than near the TMS resonance frequency.

21. Qualitatively sketch the relative changes in the spectra obtained by Fourier transformation of the second FID as compared to the first:
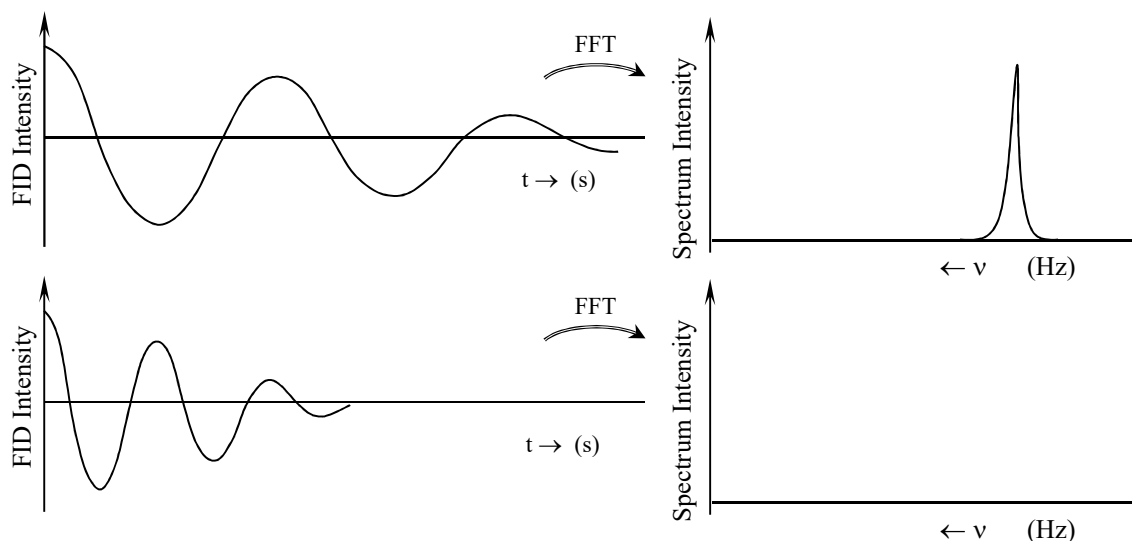

Answer: The frequency of the second FID is higher than the first (by about a factor of two). The spectrum then has the transition at higher frequency, or to the left since the frequency increases

to the left. The decay envelope of the second peak is shorter than the first; the second FID decays to zero faster than the first. As a result the FFT requires a broader range of frequencies to reproduce the time dependence. Assuming the envelopes of the FIDs are approximately exponential, e^{-t/T_2}, then the transition full width at half height in the spectrum is $1/\pi T_2$. The inverse relationship shows that quicker decays in the time domain correspond to broader transitions in the frequency spectrum. Both FIDs are initially equally intense, so to compensate for the broader transition, the second FID gives a peak with lower maximum intensity. The integral remains the same. In fact, the initial intensity of the FID gives the integrated intensity of the frequency spectrum (including multiple peaks if present).
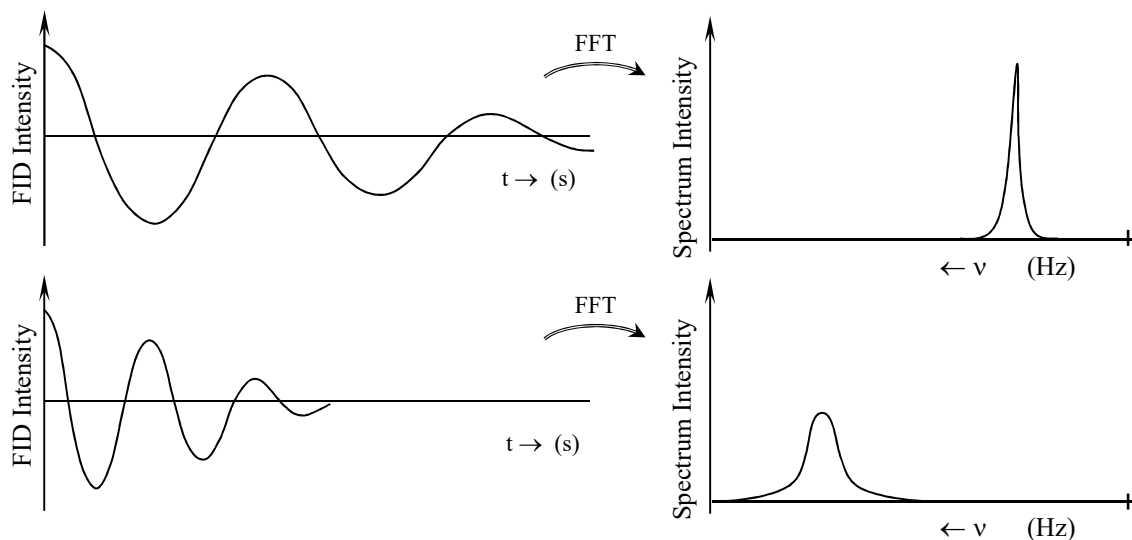
22. The inversion recovery sequence was used to determine the ^{1}H spin-lattice relaxation time of the geminal-dimethyl groups of β-ionone. The data is reproduced below. Determine the T_1. (Don't bother to get the uncertainty using linest(), just use a linear trendline.)
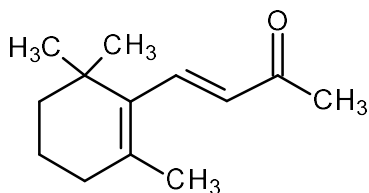

τ (s)	0.0625	0.2500	0.5000	1.000	2.000	8.000
intensity	-64.8	-32.2	4.43	47.4	76.9	84.1

Answer: The linearized form, Eq. 29.5.13, gives the slope as $-1/T_1$ of the plot of $\ln(I_o - I)$ vs. τ, where the measured transition intensity is directly proportional to the magnetization. A spreadsheet was set up to do the linear curve fit.

I_o =	84.1	
τ (s)	I	$\ln(I_o\text{-}I)$
0.0625	-64.8	5.003275
0.2500	-32.2	4.756173
0.5000	4.43	4.377893
1.0000	47.4	3.602777
2.0000	76.9	1.974081

The T_1 is then $1/1.573$ s^{-1} = 0.636 s
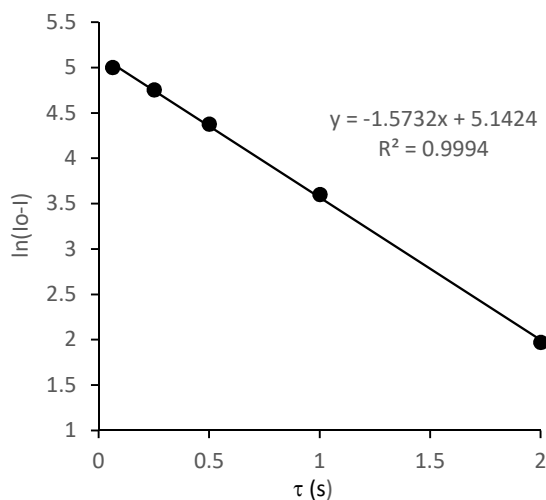
23. Derive Eq. 29.5.3 by integrating Eq. 29.5.2.

Answer: Spin-lattice relaxation is a first order kinetic process, Eq. 29.5.2:

$$\frac{dM_z}{dt} = -\frac{1}{T_1}(M_z - M_o)$$

Separating variables (*General Pattern ℘1: Simple Exponential Processes*):

$$\int_0^{M_z} \frac{dM_z}{(M_z - M_o)} = -\int_0^t \frac{1}{T_1}\,dt$$

The integrals give: $\ln(M_z - M_o)\Big|_0^{M_z} = -\frac{t}{T_1}$
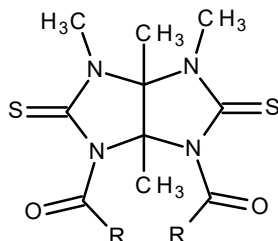
$$\ln(M_z - M_o) - \ln(-M_o) = -^t\!/T_1$$

Combining the logarithmic terms gives: $\ln\left(\dfrac{M_z - M_o}{-M_o}\right) = -^t\!/T_1$

Exponentiation of both sides of the relationships gives: $\left(\dfrac{M_z - M_o}{-M_o}\right) = e^{-^t\!/T_1}$

Cross multiplication results in: $M_z - M_o = -M_o\,e^{-^t\!/T_1}$

while solving for the magnetization gives: $M_z = M_o - M_o\,e^{-^t\!/T_1} = M_o\,(1 - e^{-^t\!/T_1})$

24. Consider the following molecule with *tert*-butyl groups as the R-groups:


Steric interactions of the *tert*-butyl groups prevent the amide bonds from being planar with each respective ring. The N-methyl groups are not equivalent, because the amide groups are twisted out of plane.[2,3] Rotation about the amide C-N bonds exchanges the chemical environment of the N-methyl resonances. Outline the experimental and data analysis steps necessary to find the activation energy of the chemical exchange process for twisting about the amide bonds.

Answer: (1). The ^{1}H-spectra of the compound are acquired as a function of temperature.
(2). Two transitions that correspond to the same proton in the two different chemical environments are determined by assigning the peaks in the spectrum. As an aid to assigning the peaks, the assumption is that at low temperature these two peaks will be equally broadened, that the peak positions approach each other with an increase in temperature, and that two peaks coalesce at high temperature.
(3). A low temperature that causes no additional peak separation and narrow transitions is used to determine the chemical shift difference between the two environments giving $\Delta v_o = v_A - v_B$. The width of the peaks is used to find the effective spin-spin relaxation time, T_{2A}' , using Eq. 29.6.4.
(4). Eqs. 29.6.4, 29.6.6, 29.6.7, 29.6.8, and 29.6.10 are used to estimate the exchange rate constant. See Example 29.6.1
(5). Based on Eq. 3.5.3, a plot of ln k vs. 1/T is constructed giving the activation energy from the slope as: slope $= - E_A/R$.

Curve fitting to the exact line shape is a preferable approach to using the approximate formulas for k. Many computer programs are available for this purpose including the "Chemical Exchange Lineshapes" applet (exchpl.html) on the course Web site or companion CD.

25. The ^{1}H-NMR spectra of the N-methyl compound shown below are plotted at -94°C and -30°C at 300 MHz. The R-groups are *tert*-butyl groups, which force the amide groups to be twisted out of plane, which makes the two N-methyl groups inequivalent. The spectra are taken in deuterated methylene chloride solution. The resonances near 3.2 ppm are the N-methyl groups. The difference in chemical shift at low temperature for the N-methyl groups is 35.0 Hz. Assume that the effective T_2' is 1.5 s. At -94°C, the full-width at half-height of one of the two N-methyl transitions in the exchanging doublet is 13.5 Hz. The coalescence temperature is -70.5°C (spectrum not shown). The width of the coalesced N-methyl peak at -30°C is 10.4 Hz. Calculate the activation energy for the twisting motion of the amide groups.[2,3]

300 MHz ^{1}H

-30°C

$(\Delta v_{1/2})^e = 10.4$ Hz

-94°C

$(\Delta v_A)^e_{1/2} = 13.5$ Hz

← δ (ppm)
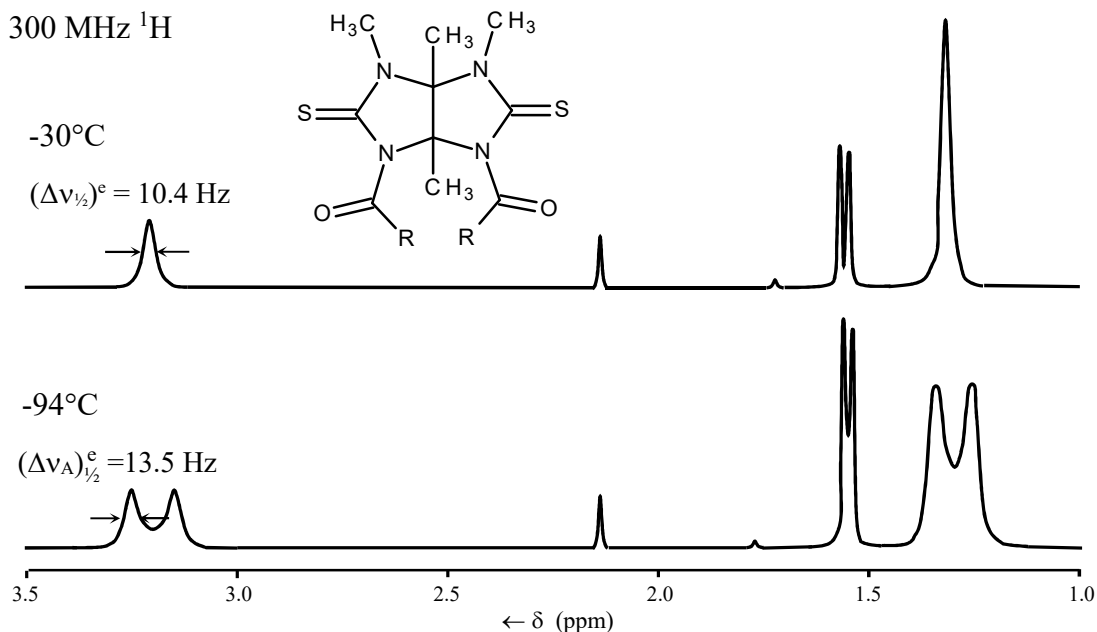
Answer: The effective T_2' of 1.5 s gives the line-width in the absence of exchange, Eq. 29.6.4:

$$(\Delta v_A)^o_{1/2} = \frac{1}{\pi T_{2A}'} = \frac{1}{\pi\ 1.5\ s} = 0.21\ \text{Hz}$$

In intermediate exchange, at -94°C, the line width is 13.5 s^{-1}, so that Eq. 29.6.6 gives:

$$k \cong \frac{\pi}{\sqrt{2}} \left[(\Delta v_A)^e_{1/2} - (\Delta v_A)^o_{1/2} \right] = \frac{\pi}{\sqrt{2}} [13.5\ s^{-1} - 0.21\ s^{-1}] = 29.5\ s^{-1}$$

At coalescence, -70.5°C, using Eq. 29.6.8: $k \cong \dfrac{\pi \Delta v_o}{\sqrt{2}} = \dfrac{\pi\ 35.0\ s^{-1}}{\sqrt{2}} = 77.8\ s^{-1}$

In fast exchange, at -30°C, the line width is 10.0 Hz, so that Eq. 29.6.10 gives:

$$k \cong \frac{\pi \Delta v_o^2}{2} \frac{1}{\left[(\Delta v_{1/2})^e - (\Delta v_A)^o_{1/2} \right]} = \frac{\pi(35.0\ s^{-1})^2}{2} \frac{1}{(10.0\ s^{-1} - 0.21\ s^{-1})} = 197\ s^{-1}$$

Assuming Arrhenius temperature dependence, Eq. 3.5.3, a plot of ln k vs. 1/T is constructed giving the activation energy from the slope as: slope = – E_A/R as shown in the following spread sheet:

T (°C)	T (K)	k (s⁻¹)	1/T (K⁻¹)	lnk
-94	179.15	29.5	0.00558	3.3844
-70.5	202.65	77.8	0.00493	4.3541
-30	243.15	197	0.00411	5.2832
slope	-1285.2	10.61	intercept	
±	104.19	0.512	±	
R^2	0.9935	0.108	s(y)	
F	152.18	1	dof	
SS$_{reg}$	1.7913	0.012	SS$_{res}$	

y = -1285.2x + 10.608
R^2 = 0.9935
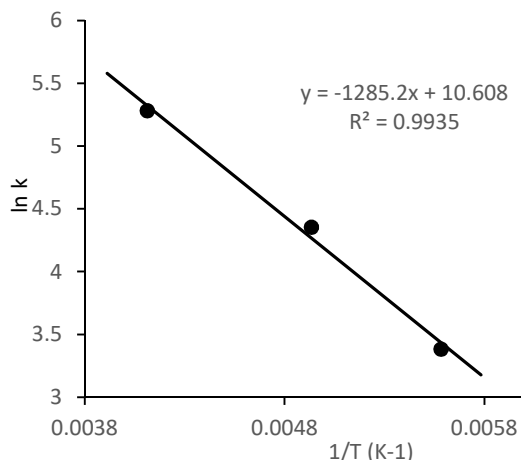
The activation energy is $E_A = - R$ slope $= 10.7$ kJ mol⁻¹.

26. Create an Excel spreadsheet to do the time averages to determine the correlation function for a random signal. Generate the random signal at equal time increments, of length Δt, with varying persistence, p:

$$f(t + \Delta t) = f(t) + (1 - p)[(2*RND() - 1) - f(t)]$$

where RND() is the built-in random number generator in Excel and $(2*RND() - 1)$ generates a random number between -1 and +1. Plot $f(t)$ and the corresponding correlation function. Hints for setting up the spreadsheet are given below. Use four different values of the persistence: p = 0, 0.2, 0.5, and 0.8. With p = 0, the signal is purely random with no correlation, $f(t + \Delta t) = (2*RND() - 1)$. Increasing p gives a signal that is increasingly slowly varying. For each value of p, generate several different plots. The results will be different in each plot; you can estimate the equivalent of the ensemble average by "averaging" the successive plots by eye. To generate each new set of random numbers, change the value in any arbitrary unused cell in the spreadsheet. Any unused cell will do, the cell chosen for generating updates shouldn't be used in the main part of the spreadsheet. From your comparison of the results for the different values of p, discuss the relationship among the persistence, the appearance of $f(t)$, and the observed approximate correlation time. You don't need to find a numerical value of the correlation time, discuss the results qualitatively.

An example spreadsheet is shown below. Only the first few rows and the final three rows are shown, to save space. Rows 2-5 and column B are input directly, that is with no formulas. The main time variable t in column B runs in 5 ns increments up to 200 ns in row 46. Seven values of the time delay, τ, are specified in row 3. The number of rows that correspond to the chosen τ value are entered in row 4. For example, 3 rows are required to give a τ delay of 15 ns.

A1	B	C	D	E	F	G	H	I	J
2	persistence=	0.5							
3		τ = (ns)	0	5	10	15	20	25	30
4		offset	0	1	2	3	4	5	6
5	t (ns)	f(t)	f(t+τ)f(t)	f(t+τ)f(t)	f(t+τ)f(t)	f(t+τ)f(t)	f(t+τ)f(t)	f(t+τ)f(t)	f(t+τ)f(t)
6	0	-0.8037	0.6459	-0.0719	-0.3803	-0.0622	-0.1562	0.2355	0.3560
7	5	0.0895	0.0080	0.0423	0.0069	0.0174	-0.0262	-0.0396	-0.0567
8	10	0.4731	0.2238	0.0366	0.0919	-0.1386	-0.2096	-0.2997	-0.3480
⋮									
46	200	0.0258	0.0007						
47									
48		average=	0.1326	0.0539	0.0126	0.0180	-0.0133	-0.0164	0.0163

The starting random value of the signal, $f(t)$, at time zero is in cell C6: =(2*RND()-1)

The subsequent value of $f(t)$ in cell C7 is: = C6 + (1-C2)*((2*RND()–1)–C6)

This formula is "filled down" to fill in the values for the remaining rows in column C. The product $f(t+\tau)\,f(t)$ is created using the Excel OFFSET function. The formula in cell D6 is:

=OFFSET($C6:$C$46,D$4,0,1,1)*$C6

The offset is specified in row 4. Make sure to set up the absolute references exactly as shown. This formula is "filled right" for columns E-J and then "filled down" for all the rows up to 46. Some cells near the bottom of the table will read "0" after filling, because there is insufficient data to complete the required calculation for long τ-values. Delete the contents of these zero cells. The averages of each column are calculated in row 48 giving $\overline{f(t+\tau)\,f(t)}$. The formula in cell D48 is: =AVERAGE(D6:D46)

This cell is "filled right" for the remaining columns E-J. Construct a plot of column C versus column B to see the time-varying random signal. Construct a plot of the averages from row 48 on the vertical axis against the τ-values on the horizontal axis.

Answer: The spreadsheet gives a different result each time. Changing the number in any cell starts the random number generator anew. You need to do several plots and then average the results by eye. In the correlation plot shown below, the solid line is added to guide the eye from point to point. With a persistence of 0.0, no correlation exists beyond the initial $C(0) = \overline{f(t)^2}$ value. In other words successive plots give results that cancel for $C(\tau)$ with $\tau > 0$. One result for a persistence of 0.5 is shown below. The $f(t)$ signal varies more slowly than with p = 0. Successive correlation function plots show an approximately exponential decay. With the persistence of 0.8, $f(t)$ varies slowly and the exponential decay of the correlation function is slow and obvious from plot to plot. Bigger persistence gives a slower decay of the correlation function and a correspondingly longer correlation time for the exponential decay.

  The method that we have chosen to generate the random signal is a poor approximation to generate a truly stationary, zero mean, random time-varying signal. However, the simple formula that we have used is sufficient for short correlation times for this simple demonstration.
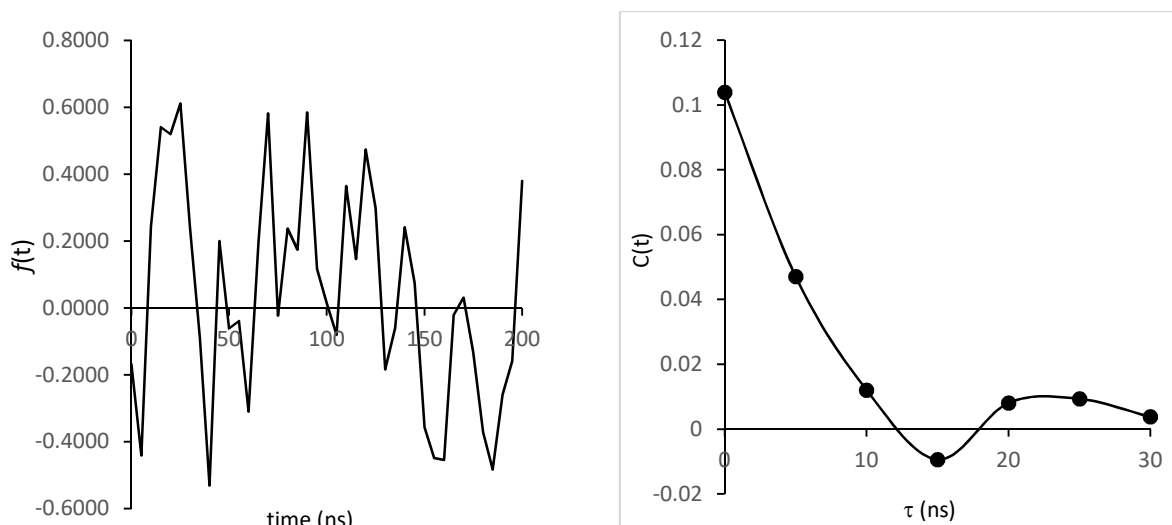
27. The benzene radical anion, $C_6H_6^-$, is produced at low temperature by reduction of benzene with an alkali metal. Predict the multiplicity of the ESR transition of the benzene radical anion and the corresponding transition intensities of the components of the multiplet.
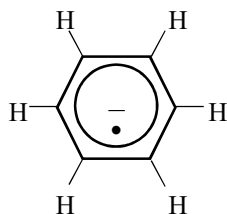
Answer: The π-system of the benzene radical anion is completely delocalized. As a result the six protons are magnetically equivalent giving a hyperfine structure multiplicity of n+1 = 7. The hyperfine coupling constant is 0.375 mT or 3.75 Gauss.[1] Using Pascal's triangle, the line intensities are 1:6:15:20:15:6:1:

| n	n+1	Multiplet				Intensity ratios					
0	1	Singlet					1				
1	2	Doublet					1	1			
2	3	Triplet				1	2	1			
3	4	Quartet			1	3	3	1			
4	5	Quintet		1	4	6	4	1			
5	6	Sextet	1	5	10	10	5	1			
6	7	Septet	1	6	15	20	15	6	1		

The derivative ESR spectrum is shown below.[1]

0.375 mT

B (T) →

28. Sketch by hand the ESR derivative spectrum of a doublet of doublets. (Note the positions of the maxima in the absorption spectrum as a basis for the zero-derivative points on the plot.)

A

B (T) →

Answer: The derivative crosses zero at the maximum of each transition. Between each transition the spectrum also has a region of zero slope. Note these zero-slope points on your initial sketch. The derivative spectrum must pass through these points:

A

$\dfrac{dA}{dB}$

B (T) →
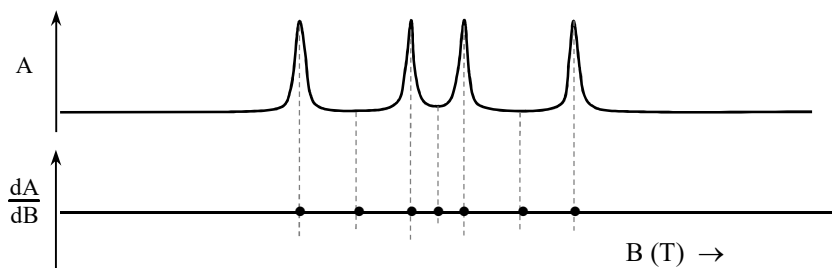
Consider starting at low field, on the left side of the spectrum. Scanning to higher field, the absorbance first increases giving a positive slope. After the maximum, the slope is negative and then zero until the next transition begins.

A

positive slope   *negative slope*

$\dfrac{dA}{dB}$

B (T) →
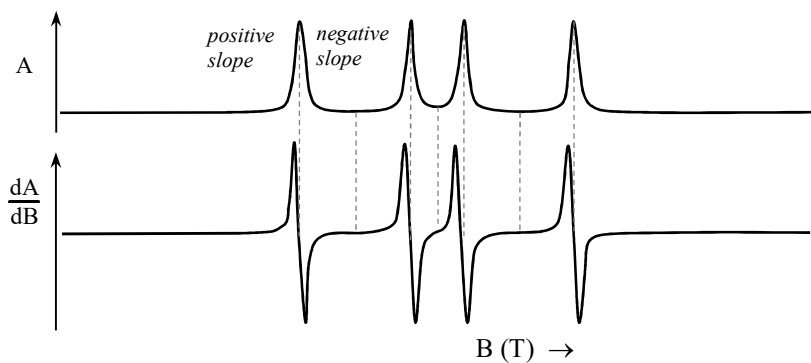
29. The π-systems of the aromatic radicals are extensively delocalized. If the nuclei of the aromatic radical lie in the x-y plane, then the π-orbitals are constructed from the overlap of $2p_z$-

orbitals. The hyperfine interaction, a_H, is then approximately proportional to the unpaired electron density in the $2p_z$-orbital at each C-H, which is denoted ρ_π:

$$a_H = Q\rho_\pi \hspace{4cm} \text{(aromatic hydrocarbons)}$$

where Q is a proportionality constant derived from model compounds. The hyperfine interaction in the benzene radical anion is 0.375 mT, or 3.75 G; assuming a π-electron density of $1/6$ gives $Q = 2.25$ mT $= 22.5$ Gauss. The hyperfine coupling constants for the naphthalene radical anion are given below. Calculate the π-electron density on the two types of ring positions.
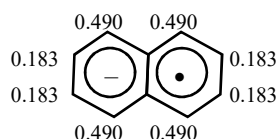

Answer: Assuming the value of Q derived from the benzene radical anion, the π-electron densities are given by $\rho_\pi = a_H/Q$:
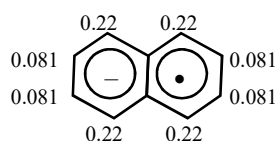
As a result the reactivity of the two types of ring positions is expected to differ. This expression is called the McConnell relationship.


30. The hyperfine coupling constants from the ESR spectra of the radical anion of toluene are shown below. The AM1 level π-molecular orbitals for toluene as a neutral molecule are also diagrammed below. Are the hyperfine constants consistent with the π-molecular orbitals? Orbital 18 is the HOMO and orbital 19 is the LUMO. (In benzene, MO 17 and 18 are degenerate and MO 19 and 20 are degenerate.)
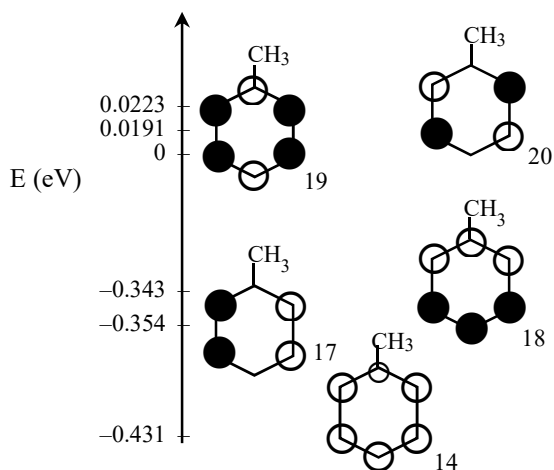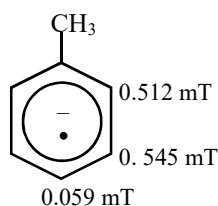
Answer: The extra electron in the anion goes into the LUMO, MO-19. MO-19 has large coefficients of the same sign on the *ortho-* and *meta-* positions. With regard to the orbital signs and coefficients, the *ortho-* and *meta-* positions are equivalent and distinct from the para-position, in agreement with the hyperfine coupling constants. You should do the AM1 calculations (we used MOPAC) for yourself to verify the molecular orbitals listed here.

31. The ESR hyperfine coupling constants and low-lying π-molecular orbitals of the toluene anion radical are given in the previous problem. Using Spartan or Gaussian, build and geometry minimize the toluene radical anion (doublet state) at the HF 3-21G level or higher. Request the molecular orbital coefficients and then generate the "radical density" or "spin density" surface for the toluene radical anion. Does this unpaired electron density surface agree with the hyperfine coupling constants and molecular orbitals listed in the previous problem? (Note that the π-molecular orbital energies depicted in the previous problem are for neutral toluene.)

Answer: The "radical density" plot generated at the HF 3-21G level using the WebMo visualization environment is shown below. Enhanced unpaired spin density resides between the *ortho-* and *meta-* positions. In other words, the largest unpaired electron spin density agrees with MO-19 shown above and the experimental hyperfine constants.
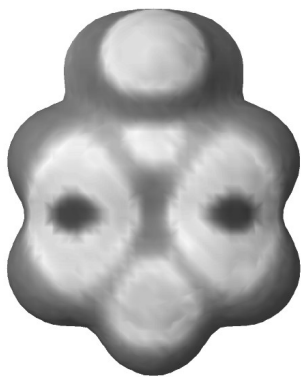
The spin density map using Spartan is even clearer, essentially reproducing the LUMO shown in the previous problem. These predictions are validated at higher theory levels including B3LYP.

32. Determine if the following statements are true or false. If the statement is false, describe the changes that are necessary to make the statement true, if possible. If the statement is true but too restrictive, give the more general statement.

(a). The spacing in ppm between the component transitions of a spin-spin splitting multiplet is constant with increasing field strength.

Answer: *False*. The spacing in Hz between the component transitions of spin-spin splitting multiplets is constant with increasing field strength. However, as the field strength increases, the resonance frequency increases, and the spacing between transitions in a multiplet decrease when expressed in ppm. A J of 9 Hz corresponds to 0.15 ppm at a 60 MHz resonance frequency and only 0.018 ppm at 500 MHz. In other words, the multiplet looks narrower at higher field when

compared to the chemical shift differences. Another way of rationalizing this effect is to note that a 10 ppm sweep width at 60 MHz is 600 Hz and the same sweep width at 500 MHz is 5000 Hz. If the spectra at 60 MHz and 500 MHz are plotted on the same size paper, then a J of 9 Hz gives transitions that appear closer together at the higher sweep width. See Problems 2 and 3 for numerical examples.

(b). In ^{1}H NMR spectroscopy, for a given spin with n-coupled neighboring protons the minimum number of transitions in the multiplet is n+1 and the maximum number of transitions in the multiplet is 2^n. Assume the given spin has a unique chemical shift.

Answer: *True*. If the n-neighbors are equivalent then an n+1 multiplet arises. If the n-neighbors are all inequivalent then the given resonance is split into a doublet by each of the n-neighbors giving 2^n transitions in the multiplet. For the n-equivalent case the multiplet transition intensities are given by Pascal's triangle. For the inequivalent case, all transitions are equally intense.

(c). The differences between the exact spin-spin splitting pattern and the first-order prediction are called second-order effects. The exact spin-spin splitting pattern is based on $J_{AB} \vec{I}_A \cdot \vec{I}_B$ while the first-order prediction is based on $J_{AB} \hat{I}_{zA} \hat{I}_{zB}$. Second-order effects are more important at high field (e.g. 500 MHz) than at low field (e.g. 60 MHz).

Answer: *False*. Even though chemical shifts in ppm are identical with increasing resonance frequency, the chemical shifts in Hz are directly proportional to field strength. As a result the difference in resonance frequency, $\nu_A - \nu_B$ in Hz, increases with resonance frequency (field strength). The important comparison for the extent of second-order effects is $\nu_A - \nu_B$ as compared to J. First-order predictions are sufficient if $|\nu_A - \nu_B| \gg J$. As a result, second-order effects are less important at high field than at low field. In fact, the simplification of spin-spin splitting patterns at high field is one of the most important advantages of high field instruments.

(d). The spin-lattice relaxation time of a given chemical environment in a ^{1}H-spectrum increases with an increase in temperature. Assume that the motion of the molecule that is most important for relaxation is <u>faster</u> than the resonance frequency at the starting temperature.

Answer: *True*. An increase in temperature increases the motions of the molecule, shortening the correlation time. However, a decrease in correlation time can either increase or decrease the relaxation rate, Figure 29.5.6. If the motion at the original temperature is faster than the resonance frequency, the correlation time is shorter than $1/2\pi\nu_o$. The initial state is to the left of the minimum in the figure. Further shortening of the correlation time moves the system further to the left, increasing the relaxation time. In other words, if motions are too fast to begin with, further increases won't improve the efficiency of the relaxation, thus increasing T_1.

(e). Consider chemical exchange between two inequivalent chemical environments. The line widths decrease with an increase in temperature if the system is initially at a temperature that is below the coalescence temperature.

Answer: *False*. As temperature increases, the exchange rate constant increases. The line widths increase with an increase in temperature if the system is starting at a temperature that is <u>below</u> the coalescence temperature. The two lines broaden until they strongly overlap with increasing exchange rate constant. At temperatures <u>higher</u> than the coalescence temperature, increases in temperature again increase the exchange rate. As the exchange rate increases the lines collapse to the average chemical shift and the line width decreases to the extreme narrowing limit.

Literature Cited:

1. A. Carrington, A. D. McLachlan, *Introduction to Magnetic Resonance*, Harper & Row, New York, NY, 1967. pp. 182-194.

2. A. D. Bain, "Chemical Exchange NMR," *Prog. in N.M.R*., **2003**, *43*, 67-71.

3. P. A. Duspara, C. F. Matta, S. I. Jenkins, P. H. M. Harrison, "Twisted Amides: Synthesis and Structure of 1,6-Dipivaloyl-3,4,7,8- tetramethyl-2,5-dithioglycoluril," *Org. Lett.*, **2001**, *3(4)*, 495–498.