Chapter 4: Kinetic Mechanisms

How can a reaction be first-order? A unimolecular elementary step implies that no collision occurs; how can the reactant overcome its activation energy barrier?

The primary goal of chemical kinetic studies is to determine, as completely as possible, the mechanism for a chemical reaction. A mechanism is postulated based on the available experimental evidence. The empirical rate law, activation energies, pre-exponential factors, equilibrium constants, and the presence of any intermediates are the key pieces of information that help to construct plausible mechanisms. A proposed mechanism must agree with the empirical rate law. In other words, a proposed mechanism should result in a predicted overall rate law that has the same concentration and time dependences as the experimentally determined rate law. However, the empirical rate law for a reaction far from equilibrium may be different from the rate law for the same reaction close to equilibrium. In addition, the form of the rate law may depend on the overall pressure for a gas-phase reaction or the initial concentrations in solution. The corresponding mechanisms may then be different far from equilibrium or near equilibrium, or at high and low overall pressure or concentration. A proposed **complete mechanism** should explain such shifts in the observed rate law and be applicable over a range of initial conditions and at equilibrium.

The rate law for each elementary step is directly determined from the molecularity, because the elementary steps describe the collisions that take place. The steps in the mechanism should add to give the overall reaction stoichiometry. However, there may be more than one mechanism that agrees with the experimental rate law. Such alternate mechanisms are called **kinetically equivalent**. One way to distinguish between kinetically equivalent mechanisms is to identify intermediates in the reaction.

Intermediates: An **intermediate** is a species that is neither a reactant nor a product. **Reactive intermediates** react quickly after formation and so never build to significant concentrations. The concentration of **stable intermediates**, on the other hand, can build to a sizable fraction of the concentration of the original reactants. If the rate law for a reaction involves the concentration of an intermediate, the mechanism cannot consist of a single elementary process. Typical intermediate species include reactive atoms, free radicals, and charged species such as carbocations and carbanions. Identifying intermediates experimentally is helpful in postulating a mechanism for the reaction.

Building a Plausible Mechanism: A good example, from the beginning of Chapter 3, is the $H_2 + I_2 \rightarrow 2\ HI$ reaction. The empirically determined rate law is:

$$\upsilon = \frac{d[HI]}{dt} = k\,[H_2][I_2]$$

The direct molecular collision of H_2 and I_2 molecules is one possible mechanism that agrees with the empirical rate law. This direct molecular mechanism occurs in one elementary step, $H_2 + I_2 \rightarrow 2\ HI$. The fact that the experimental rate law follows the stoichiometry of the reaction does not imply that the process occurs in one elementary step. Conversely, however, if the empirical rate law does not follow the reaction stoichiometry, the reaction cannot be a single elementary step. One alternate mechanism type for this reaction is the pre-equilibrium mechanism:

$$I_2\,(g) \underset{k_{-1}}{\overset{k_1}{\rightleftarrows}} 2I\,(g) \qquad \text{(rapid bidirectional)}$$

$$H_2\,(g) + 2\,I\,(g) \xrightarrow{k_3} 2HI\,(g) \qquad \text{(slow, } k_3 << k_1,\ k_{-1}\text{)}$$

where the I atoms are reactive intermediates. We follow the convention that reaction arrows are given in bold to identify an elementary step. The rate law for the formation of product is:

$$\upsilon = \frac{1}{2}\frac{d[HI]}{dt} = k_3\,[H_2][I]^2$$

as dictated by the molecularity of the second step. If the first reversible step is rapid in both directions compared to the second step, k_1 and $k_{-1} >> k_3$, the reversible step is near equilibrium, and then:

$$K_c = \frac{k_1}{k_{-1}} = \frac{[I]^2}{[I_2]}$$

Solving for the concentration for the I atoms gives $[I]^2 = K_c\,[I_2]$, which when substituted into the rate law for the formation of products gives:

$$\upsilon = \frac{1}{2}\frac{d[HI]}{dt} = k_3\,K_c\,[H_2][I_2]$$

This rate law and the direct single-step mechanism both agree with the empirical rate law. Proving that a mechanism is the correct mechanism is often compared to building a case in a court of law, “beyond a shadow of a doubt.” You can build evidence for a proposal, but you can never be sure that the proposal is absolutely and exclusively correct. One way to build evidence for a mechanism is to detect the proposed intermediates. Even better is to measure the time course for the intermediates and show that the integrated rate laws for the intermediates agree with the experimental data. Another way is to compare the predicted activation energy for the different proposals to the experimental activation energy. For the $H_2 + I_2$ reaction, the activation energy for the direct mechanism is predicted to be lower than the activation energy for the pre-equilibrium mechanism, which must break the I-I bond.[1] Therefore, the direct molecular mechanism is presumed to be more important at low temperatures. In the end, there may be several competing mechanisms that are active under different experimental circumstances, or even several competing mechanisms that are always active. As a consequence there are few complete mechanisms that are widely accepted.

In this chapter we discuss some simple typical mechanisms and the corresponding overall rate laws and integrated rate expressions. We then apply the methods that we develop for simple mechanisms to chain reactions. We also discuss the restrictions that are placed on valid mechanisms when the reaction is close to equilibrium.

4.1 A Mechanism is a Sequence of Elementary Steps

One of the best ways to learn about reaction mechanisms is to consider simple examples. The examples we cover will highlight the most common aspects of all reaction mechanisms, no matter how complex. Complex reactions are combinations of these simpler examples. We also want to develop general insight into the effect of a given mechanism on the overall rate law. General insight is important as you consider important environmental and biological examples.

The current model for tropospheric ozone production considers 70 species and about 140 mechanistic steps. To unravel such important and complex mechanisms it is helpful to be able to conceptually break the complex mechanism into simpler components that you can relate to simplified rate laws. The first mechanism we consider is a reaction that has two parallel steps. Then we will consider a mechanism with two sequential steps. Finally, we consider several examples of the reversible first-step mechanism.

Parallel Mechanism – Competitive Reactions: First we consider a single reactant that can form two products. Two reactions from the same reactant are said to be in parallel. The two parallel steps compete with each other for the available reactant, so this mechanism is often called a competitive mechanism. To simplify matters we consider two parallel steps that are given by:

$$\mathrm{A} \xrightarrow{k_1} \mathrm{B}$$

$$\mathrm{A} \xrightarrow{k_2} \mathrm{C} \qquad \text{(1st-order parallel unidirectional)} \qquad 4.1.1$$

The two parallel steps are also sometimes called **channels**. The reactant disappears through two channels in this mechanism. Because these two steps are postulated to be elementary processes, the rate law for each elementary process is determined by the molecularity. Each parallel step is unimolecular and the rate law for the disappearance of A is first order in A for both steps:

$$-\frac{d[\mathrm{A}]}{dt} = k_1[\mathrm{A}] + k_2[\mathrm{A}] \qquad \text{(1st-order parallel unidirectional)} \qquad 4.1.2$$

You can read this rate law as saying the disappearance of A is through the first-order decomposition of A to produce B and the simultaneous first-order decomposition of A to produce C. Both parallel steps decrease the concentration of A. Since [A] is a common factor in the rate law, we can define the combined rate constant, k, as:

$$k \equiv k_1 + k_2 \qquad \text{(1st-order parallel unidirectional)} \qquad 4.1.3$$

Then, the rate law reduces to a simple first-order process:

$$-\frac{d[\mathrm{A}]}{dt} = k\,[\mathrm{A}] \qquad \text{(1st-order parallel unidirectional)} \qquad 4.1.4$$

In other words, the rate of disappearance of the reactant doesn't depend on what the products are. Eq. 4.1.4 is a *simple exponential process*, and using general pattern ***℘1*** or equivalently Eq. 3.2.5:

$$[\mathrm{A}] = [\mathrm{A}]_o\, e^{-kt} = [\mathrm{A}]_o\, e^{-(k_1+k_2)\,t} \qquad \text{(1st-order parallel unidirectional)} \qquad 4.1.5$$

giving simple first-order decay. We can substitute this result for the time dependence of A into the rate laws for the formation of B and C:

$$\frac{d[\mathrm{B}]}{dt} = k_1\,[\mathrm{A}] = k_1\,[\mathrm{A}]_o\, e^{-(k_1+k_2)\,t} \qquad \text{(1st-order parallel unidirectional)} \qquad 4.1.6$$

$$\frac{d[\mathrm{C}]}{dt} = k_2\,[\mathrm{A}] = k_2\,[\mathrm{A}]_o\, e^{-(k_1+k_2)\,t} \qquad \text{(1st-order parallel unidirectional)} \qquad 4.1.7$$

The general form of the indefinite integral is $\int e^{ax}\,dx = \frac{1}{a}e^{ax}$, giving the integral of Eq. 4.1.6 as:

$$[B] = -\frac{k_1[A]_o}{(k_1+k_2)}e^{-(k_1+k_2)t} + c \qquad \text{(1st-order parallel unidirectional)} \qquad 4.1.8$$

The boundary condition at t = 0 is $[B]_o = 0$ and Eq. 4.1.8 becomes:

$$0 = -\frac{k_1[A]_o}{(k_1+k_2)} + c \qquad \text{(1st-order parallel unidirectional)} \qquad 4.1.9$$

Solving for the integration constant:

$$c = \frac{k_1[A]_o}{(k_1+k_2)} \qquad \text{(1st-order parallel unidirectional)} \qquad 4.1.10$$

Substitution of the integration constant back into Eq. 4.1.8 and collecting common factors gives:

$$[B] = \frac{k_1[A]_o}{(k_1+k_2)}\left(1 - e^{-(k_1+k_2)t}\right) \qquad \text{(1st-order parallel unidirectional)} \qquad 4.1.11$$

Eqs. 4.1.6 and 4.1.7 are in exactly the same form, except for the specific rate constant. Then by analogy for C we find:

$$[C] = \frac{k_2[A]_o}{(k_1+k_2)}\left(1 - e^{-(k_1+k_2)t}\right) \qquad \text{(1st-order parallel unidirectional)} \qquad 4.1.12$$

A typical plot of Eqs. 4.1.5, 4.1.11, and 4.1.12 is shown in Figure 4.1.1.

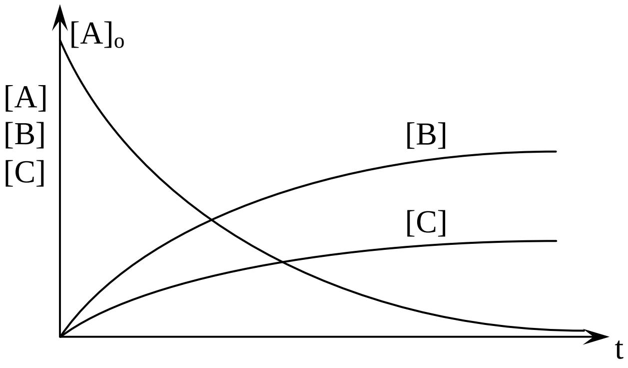


Figure 4.1.1: Competitive first-order mechanism.

Notice if the second parallel step is negligible, then $k_2 \approx 0$, and Eqs. 4.1.5 and 4.1.11 reduce to the expressions for a simple single-step first-order reaction with rate constant k_1. Also, Eq. 4.1.12 correspondingly gives $[C] \approx 0$. Another way to look at this mechanism is to find the ratio of the concentrations of the two products at any time during the course of the reaction by dividing Eq. 4.1.11 by 4.1.12:

$$\frac{[B]}{[C]} = \frac{k_1}{k_2} \qquad \text{(1st-order parallel unidirectional)} \qquad 4.1.13$$

Notice that the ratio of the two products does not depend on time. The relative yield of B is the same at both early stages and late stages of the reaction. This general feature of this mechanism is often exploited by organic chemists for studies of competitive processes. A common lab experiment in Organic Chemistry courses that you may have done is the competitive S_N1 reaction of *tert*-butyl alcohol with chloride and bromide using an acid catalyst.

A useful generalization of this mechanism is based on reaction lifetimes (review Sec. 3.2). If the first reaction A→B occurs alone, the reaction lifetime, or 1/e time, is $\tau_1 = 1/k_1$. If the second reaction A→C occurs alone, the reaction lifetime is $\tau_2 = 1/k_2$. The lifetime of the reaction when both steps occur is given from Eqs. 4.1.3 and 4.1.5:

$$\frac{1}{\tau_{obs}} = \frac{1}{\tau_1} + \frac{1}{\tau_2} = k_1 + k_2 \qquad \text{(1st-order parallel unidirectional)} \qquad 4.1.14$$

where τ_{obs} is the observed lifetime for the reaction and $\tau_{obs} = 1/k$. One easy way to remember this equation is to compare this result to electronic circuits. Two resistors, R_1 and R_2, in a parallel connection give the overall resistance, $1/R = 1/R_1 + 1/R_2$. Parallel chemical reactions and parallel resistors have analogous behavior.

What happens if there are more than two parallel mechanistic steps? For the disappearance of the reactant, the rate constants for each parallel step add to give the rate constant for the disappearance of A and the overall life time is given by:

$$\frac{1}{\tau_{obs}} = \frac{1}{\tau_1} + \frac{1}{\tau_2} + \frac{1}{\tau_3} + \ldots. \qquad \text{(multiple 1st-order parallel unidirectional)} \qquad 4.1.15$$

Example 4.1.1: *Parallel Mechanisms and Reaction Life Times*
In an acid catalyzed reaction of *tert*-butyl alcohol with excesses of tetramethylammonium chloride and bromide, the pseudo first-order rate constant for the formation of *tert*-butylchloride is 0.112 min^{-1} and for *tert*-butylbromide is 0.332 min^{-1}. Calculate the observed lifetime for the decomposition of *tert*-butyl alcohol. Comment on the observed, overall lifetime as compared to the lifetimes of the single product reactions.

Answer: The overall rate constant for the disappearance of the reactant is:

$$k = 0.112\ \text{min}^{-1} + 0.332\ \text{min}^{-1} = 0.444\ \text{min}^{-1}$$

The lifetime for the single-product reaction with chloride alone is: $\tau_{Cl} = 1/k_{Cl} = 8.93$ min

and for bromide alone is: $\tau_{Br} = 1/k_{Br} = 3.01$ min

The lifetime for the overall, observed reaction is calculated from:

$$\frac{1}{\tau_{obs}} = \frac{1}{\tau_{Cl}} + \frac{1}{\tau_{Br}} = \frac{1}{8.93\ \text{min}} + \frac{1}{3.01\ \text{min}} = 0.444\ \text{min}^{-1}$$

or $\tau_{obs} = 2.25$ min. The observed lifetime is closest to the lifetime of the fastest reaction. The fastest channel has the shortest lifetime and dominates the observed lifetime.

Up to this point we have only considered unidirectional steps for a parallel mechanism. However, if both parallel processes are reversible there will be competition based on thermodynamic versus kinetic control of the reaction. Thermodynamic versus kinetic stability is a key concept in chemistry. For complex mechanisms it is often very helpful, or even necessary, to use purely numerical methods to find the time course that is predicted by a mechanism. The reversible, parallel mechanism is a good example.

The Finite Difference Approximation Allows the Numerical Integration of Rate Laws: Integrated rate laws rapidly increase in complexity as the mechanisms require more steps. For most complex mechanisms, the integrated rate laws cannot be determined analytically. Numerical approximations are very useful for finding the corresponding time course by integrating the rate laws. For example, the first-order rate law is:

$$-\frac{d[A]}{dt} = k\,[A] \qquad 4.1.16$$

Approximating the derivative with a finite difference gives:

$$-\frac{\Delta[A]}{\Delta t} = k\,[A] \qquad (\Delta t << 1/k) \qquad 4.1.17$$

Multiplying by –Δt gives:

$$\Delta[A] = -\,k\,[A]\,\Delta t \qquad (\Delta t << 1/k) \qquad 4.1.18$$

The time course is divided into equal time intervals, Δt. If the concentration at time t is [A](t), and the concentration in the next time interval is [A](t + Δt), then the concentration difference over the time interval, Δ[A], as given in Eq. 4.1.18 corresponds to:

$$\Delta[A] = [A](t + \Delta t) - [A](t) \qquad 4.1.19$$

Substituting this last equation into the finite difference approximation, Eq. 4.1.18, with some rearrangement gives:

$$[A](t + \Delta t) = [A](t) - k\,[A](t)\,\Delta t \qquad (\Delta t << 1/k) \qquad 4.1.20$$

Starting with the initial value $[A]_o$, this equation is applied repeatedly to generate each step for the time course. The trick to finding adequately accurate results using the finite difference approach is to use a Δt that is small enough. Setting Δt << 1/k is a commonly used guideline. In practice, you should run your calculation with two different Δt values and compare. If the runs differ significantly, choose an even smaller Δt and run again. You can do finite difference integration using Excel. Table 4.1.1 is an example Excel spreadsheet based on Eq. 4.1.20 to show how well the finite difference approximation works.

There are also a wide variety of computer applications for finite difference integration of differential equations. Computer-based algebra programs like *Maple* and *Mathematica* and numerical simulation programs like *MathCad*, *MatLab*, and *Stella* have a well developed suite of routines for differential equations. The "Kinetics Mechanism Simulation" applet on the textbook Web site and on the companion CD is specific to chemical kinetics. However, this applet is simplified for Web delivery and therefore less accurate.

All these applications use sophisticated numerical approximation techniques to decrease the errors inherent in finite difference integration, but the form of Eq. 4.1.20 is still the basis.

Table 4.1.1. Comparison of the finite difference approximation to the exact solution of a first-order rate law. $k=0.3\ s^{-1}$ and $\Delta t = 0.1$ s. Closer agreement is obtained with smaller Δt.

t	[A]=[A]-k[A]Δt	exact
0	1.000	1.000
0.1	0.970	0.970
0.2	0.941	0.942
0.3	0.913	0.914
0.4	0.885	0.887
0.5	0.859	0.861
0.6	0.833	0.835
0.7	0.808	0.811
0.8	0.784	0.787
0.9	0.760	0.763
1	0.737	0.741

The Kinetic Product Dominates Early in the Progress of a Reversible Parallel Reaction: Consider a first-order parallel mechanism with reversible steps and products X and Y:

$$A \underset{k_{-1}}{\overset{k_1}{\rightleftarrows}} X \tag{4.1.21}$$

$$A \underset{k_{-2}}{\overset{k_2}{\rightleftarrows}} Y \tag{4.1.22}$$

X and Y compete for A with equilibrium constants $K_{c1} = k_1/k_{-1}$ and $K_{c2} = k_2/k_{-2}$ for the two steps. The thermodynamic product is the product with the larger equilibrium constant. The kinetic product is the result of the faster step, which has the larger forward rate constant. The rate law for A is given by:

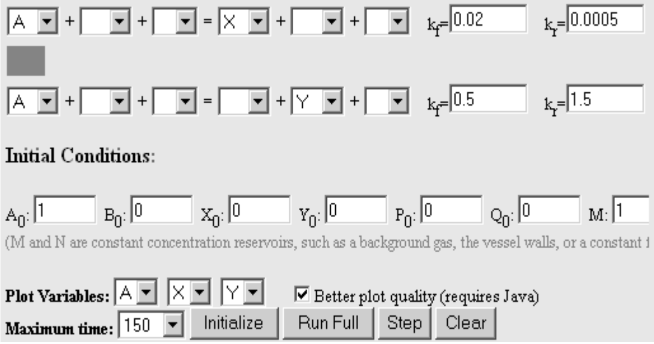
$$\frac{d[A]}{dt} = -k_1[A] + k_{-1}[X] - k_2[A] + k_{-2}[Y] \tag{4.1.23}$$

The integrated rate law for this mechanism would be complex (see Problem 5.36 and 5.37). Finite difference integration is useful for helping to understand the relationships, Figure 4.1.2.

Example 4.1.2: *Kinetic vs. Thermodynamic Control*
Numerically integrate the rate laws for the mechanism in Eqs. 4.1.21 and 4.1.22 using the rate constants $k_1 = 0.020\ s^{-1}$, $k_{-1} = 0.00050\ s^{-1}$, $k_2 = 0.50\ s^{-1}$, and $k_{-2} = 1.50\ s^{-1}$. Determine the equilibrium constants for the formation of each product. Determine how to optimize the yield of each of the products.

Answer: The equilibrium constant for the first step is $K_{c1} = 40$ and for the second step is $K_{c2} = 0.333$. So the thermodynamically favored product is X. The rate constant for the formation of Y, however, is 25 times larger than for X.

www For the numerical integration we used the “Kinetics Mechanism Simulator” applet. Here’s how the values were entered:



A + + = X + + k_f=0.02 k_r=0.0005

A + + = + Y + k_f=0.5 k_r=1.5

Initial Conditions:

A_0: 1 B_0: 0 X_0: 0 Y_0: 0 P_0: 0 Q_0: 0 M: 1

(M and N are constant concentration reservoirs, such as a background gas, the vessel walls, or a constant

Plot Variables: A X Y ☑ Better plot quality (requires Java)

Maximum time: 150

Figure 4.1.2 gives the simulation results:

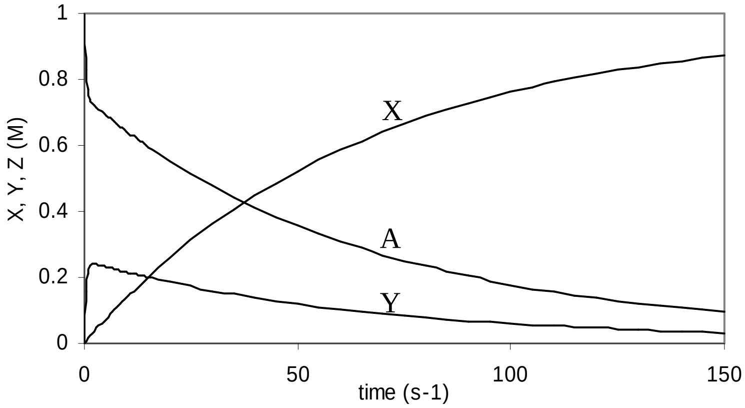


Figure 4.1.2: Kinetic versus thermodynamic control.

The kinetic product is favored at short times and the thermodynamic product is favored at long times. In the applet the default Δt is determined by the maximum time chosen and number of steps as Δt = max time/750. The simulation was additionally run for shorter maximum times to decrease Δt and no significant differences were found, thus validating the accuracy.

We now consider what happens if we have two mechanistic steps in a consecutive sequence. *Consecutive Reactions – Reactive Intermediates*: One of the most common simple mechanisms is the formation of an intermediate, B, that subsequently reacts to form products:

$$A \xrightarrow{k_1} B \xrightarrow{k_1'} C \qquad \text{(1st-order consecutive unidirectional)} \qquad 4.1.24$$

These two reaction steps are **consecutive**. Before we solve for the integrated rate laws, we should take a moment to qualitatively predict the expected time course for B. For a **reactive intermediate** the first step is relatively difficult, but after the formation of the intermediate, the second step is rapid. In this case, we find $k_1 << k_1'$. The concentration of the reactive intermediate is predicted to remain low during the course of the reaction because the intermediate reacts quickly after formation. On the other hand, if the first step is intrinsically rapid and the second step is slow, $k_1 >> k_1'$, the concentration of the intermediate builds to large levels before the second slow step removes the intermediate. In other words, in this case B is a **stable intermediate**. Let's see if our expectations are met by the rate laws based on Eq. 4.1.24.

Assuming each of these steps is an elementary process, each step is unimolecular and the corresponding first-order rate laws are:

$$\frac{d[A]}{dt} = -k_1[A] \qquad \text{(1st-order unidirectional)} \qquad 4.1.25$$

$$\frac{d[B]}{dt} = k_1[A] - k_1'[B] \qquad \text{(1st-order consecutive unidirectional)} \qquad 4.1.26$$

$$\frac{d[C]}{dt} = k_1'[B] \qquad \text{(1st-order consecutive unidirectional)} \qquad 4.1.27$$

The rate law for the concentration of B has two terms: the first for the rate of appearance of B by the first step in the mechanism and the second for the disappearance of B to form product. The sign of the first term is positive because the first step produces B, increasing [B]. The second term has a negative sign because the second step removes B to produce products, decreasing [B].

The integrated rate law for the concentration of A from Eq. 4.1.25 is just simple first-order decay, and from Eq. 3.2.5:

$$[A] = [A]_o\, e^{-k_1 t} \qquad \text{(1st-order unidirectional)} \qquad 4.1.28$$

Substitution of this equation for [A] into the rate law for B, Eq. 4.1.26, and integration using standard integral tables gives (see Example 6.3.2 and 6.3.3):

$$[B] = [A]_o\left(\frac{k_1}{k_1'-k_1}\right)\left(e^{-k_1 t} - e^{-k_1' t}\right) \qquad \text{(1st-order consecutive unidirectional)} \quad 4.1.29$$

Using Eqs. 4.1.28 and 4.1.29, we can solve for the time dependence of C by difference using the mass balance equation:

$$[A]_o = [A] + [B] + [C] \qquad 4.1.30$$

Solving 4.1.30 for [C] and substituting 4.1.28 for [A] and 4.1.29 for [B] gives:

$$[C] = [A]_o\left[1 + \left(\frac{1}{k_1 - k_1'}\right)\left(k_1'\, e^{-k_1 t} - k_1\, e^{-k_1' t}\right)\right]$$

(1st-order consecutive unidirectional) 4.1.31

One way to become comfortable with any complicated expression is to plot the behavior for typical values of the rate constants. Consider the case when the first step has the smaller rate constant, $k_1 << k_1'$, Figure 4.1.3. Notice that A decays by a simple first-order process. The time course of A doesn't depend on what happens in later steps. For very short times, B builds up in a roughly first-order process. This initial short time period when B is rapidly increasing and C increases slowly is sometimes called an induction period. However, after the concentration of B reaches a small but significant value, the rate of production of product increases limiting any further increase in B. When $k_1 << k_1'$, B corresponds to a reactive intermediate and the concentration of B remains small during the course of the reaction as we expected.

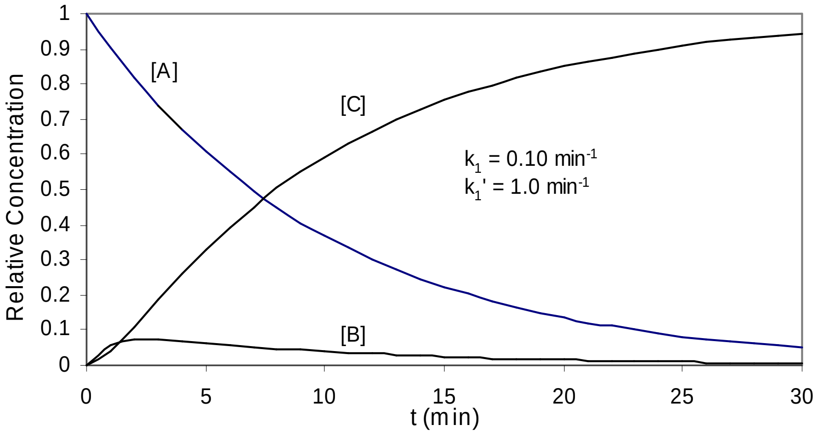


Figure 4.1.3: Consecutive reaction mechanism with $k_1 = 0.10\ \text{min}^{-1}$ and $k_1' = 1.0\ \text{min}^{-1}$. The first step is the slow step.

As we have noted before, another way to become comfortable with a complicated expression is to look at the expression in the limit that corresponds to a mechanism that we already know. If the first step has the smaller rate constant, $k_1 << k_1'$, Eq. 4.1.31 reduces to:

$$[C] = [A]_o (1 - e^{-k_1 t}) \qquad (k_1 << k_1') \qquad 4.1.32$$

This last result is identical to the result for a simple one-step first-order process, Eq. 3.2.8. The most important and interesting point is that even though C is formed in the step with rate constant k_1', the rate constant that appears in Eq. 4.1.32 is the slower rate constant k_1. In other words, the intrinsically slow step is the **rate determining step**. The rate of formation of product, after the induction period, is only dependent on the rate of the first step. You will explore the case when $k_1 >> k_1'$ in your homework (Problem 7). However, we can anticipate that when the second step is the intrinsically slow step the concentration of the intermediate builds to significantly higher levels than shown in Figure 4.1.3.

A careful look at Figure 4.1.3 provides a very useful insight that we can generalize to give a method for simplifying and approximating complex rate laws. The concentration of the intermediate is remarkably constant for most of the course of the reaction. This observation is the basis for the steady-state approximation. While it is always possible to use computer-based algorithms to numerically integrate the rate laws for complex mechanisms, it is useful to be able to develop an approximate method that helps us to develop our chemical insight.

4.2 The Steady-State Approximation Simplifies Rate Laws

Consecutive Reactions: For the consecutive reaction mechanism, when the first step is the intrinsically slow step, the concentration of the intermediate is constant during most of the time course of the reaction. For the short-time interval at the beginning of the reaction, the concentration of B must increase from zero. However, once the concentration of B is large enough, the rate of the second step increases until the second step has the same rate as the first step. After the induction period, then, the time derivative of this approximately constant concentration of the intermediate goes to zero:

$$\frac{d[B]}{dt} = k_1[A] - k_1'[B] \approx 0 \qquad (k_1 << k_1') \qquad 4.2.1$$

This approximation is called the **steady-state approximation**. Adding k_1' [B] to both sides of the second equality in Eq. 4.2.1 shows that the rates of the two steps are equal when the steady-state approximation is valid:

$$k_1[A] = k_1'[B] \qquad (k_1 << k_1') \qquad 4.2.2$$

Notice that equalities in Eqs. 4.2.1 and 4.2.2 do not imply that the concentration of B is zero; they only require that the concentration of B, big or small, remains constant. The steady-state approximation is useful because it allows the concentrations of reactive intermediates to be calculated. For example, we can solve Eq. 4.2.2 for [B]:

$$[B] = \frac{k_1}{k_1'}[A] \qquad (k_1 << k_1') \qquad 4.2.3$$

This value for the intermediate concentration can then be substituted into the rate law for the production of product, Eq. 4.1.27, to give:

$$\frac{d[C]}{dt} = k_1' \frac{k_1}{k_1'}[A] = k_1[A] \qquad (k_1 << k_1') \qquad 4.2.4$$

Notice something quite striking. The rate constant that now appears in the rate law for the production of product is the rate constant for the first, intrinsically slower step. In other words, the intrinsically slow step is the rate determining step, just as we discovered from Eq. 4.1.32. In fact, substituting the concentration of A from Eq. 4.1.28 into Eq. 4.2.4 gives:

$$\frac{d[C]}{dt} = k_1[A]_o\, e^{-k_1 t} \qquad (k_1 << k_1') \qquad 4.2.5$$

which when integrated gives Eq. 4.1.32. The steady-state approximation is a "short cut" that avoids having to do the exact integrals of all the coupled steps in a multi-step mechanism. Remember, however, that the steady-state approximation does not hold at the beginning of the reaction. However, all net rates for each reversible step do approach zero as the reaction approaches equilibrium; the steady-state approximation becomes exact at equilibrium.

The steady-state approximation can be used in a wide variety of situations to help simplify complex rate laws. Our goal now is to show you how to apply the steady-state approximation to a variety of important problems. In each case the steady-state approximation allows the concentrations of reactive intermediates to be calculated, which can then be used to simplify the rate law for the production of products. The three examples we discuss next are nucleophilic substitution, the Michaelis-Menten enzyme mechanism, and the mechanism for first-order gas-phase reactions. Each of these examples uses the same two-step mechanism that has a reversible first step. The examples really differ only in the stoichiometry.

Reversible First-Step Mechanism — Nucleophilic Substitution, S_N1: The different reaction mechanisms that you learned in Organic Chemistry are differentiated by their experimentally determined rate laws. The S_N1 mechanism is an important example. Consider a reactant with a good leaving group, X, which reacts by a reversible first-order process to form a reactive carbocation intermediate, R^+. Subsequent to the initial reversible step, the reactive intermediate reacts with a nucleophile, Nuc:$^-$, to form the product:

$$RX \underset{k_{-1}}{\overset{k_1}{\rightleftarrows}} R^+ + X^- \qquad 4.2.6$$

$$R^+ + Nuc{:}^- \xrightarrow{k_2} R\text{-}Nuc \qquad 4.2.7$$

If R-X is R-Cl, R-Br, or R-I, then X^- is simply Cl^-, Br^-, or I^-. The nucleophile is often OH^- or is derived from an alcohol. For notational simplicity we represent the product as P, where [P] = [R-Nuc]. The rate law for the formation of product is:

$$\frac{dP}{dt} = k_2\,[R^+]\,[Nuc{:}^-] \qquad 4.2.8$$

This rate law looks quite straight forward until you realize that it includes the concentration of the reactive intermediate. The concentration of the reactive intermediate is often very difficult to measure or even detect. There are two approaches to approximations at this point. We can assume a pre-equilibrium mechanism or use the steady-state approximation. We start with the pre-equilibrium mechanism.

Use the Pre-equilibrium Approximation When k_2 is Small: If $k_2 << k_1$ and k_{-1}, then the initial reversible steps can essentially come to equilibrium at each point during the reaction. The rate law for the rate based on the concentration of the reactant is given by:

$$\frac{d[RX]}{dt} = -\,k_1\,[RX] + k_{-1}\,[R^+]\,[X^-] = 0 \qquad (k_2 << k_1, k_{-1}) \quad 4.2.9$$

At equilibrium the net rate is zero and the ratio of the products to reactants is then given by the equilibrium constant:

$$K_c = \frac{k_1}{k_{-1}} = \frac{[R^+]\,[X^-]}{[RX]} \qquad (k_2 << k_1, k_{-1}) \quad 4.2.10$$

Solving for the concentration of the reactive intermediate gives:

$$[R^+] = K_c\,\frac{[RX]}{[X^-]} \qquad (k_2 << k_1, k_{-1}) \quad 4.2.11$$

Substitution of this approximation for the reactive intermediate into the rate law for the formation of product, Eq. 4.2.8, gives:

$$\frac{d[P]}{dt} = \frac{k_2\,K_c\,[RX]\,[Nuc{:}^-]}{[X^-]} \qquad (k_2 << k_1, k_{-1}) \quad 4.2.12$$

Notice that $[X^-]$ appears in the denominator, so $[X^-]$ is an inhibitor. Also note the observed rate constant for this mechanism is actually a product of a rate constant and an equilibrium constant, that is $k_{obs} = k_2\,K_c$.

The pre-equilibrium mechanism is common. However, this approximation is quite restrictive. The intrinsic rate of the second step must be much slower than that of the reversible steps. In this S_N1 case, in contrast, once the reactive intermediate forms, we expect subsequent steps to proceed with a large rate constant. So the pre-equilibrium mechanism may not be a realistic approximation. This is the point where the steady-state approximation comes to the rescue.

S_N1 General Case with the Steady-state approximation: To use the steady-state approximation we focus on the rate law for the reactive intermediate:

$$\frac{d[R^+]}{dt} = k_1 [RX] - k_{-1} [R^+] [X^-] - k_2 [R^+] [Nuc{:}^-] = 0 \qquad (k_1 << k_2) \qquad 4.2.13$$

Once again the steady-state approximation allows the concentration of the reactive intermediate to be approximated. Solving Eq. 4.2.13 for $[R^+]$:

$$[R^+] = \frac{k_1 [RX]}{k_{-1} [X^-] + k_2 [Nuc{:}^-]} \qquad (k_1 << k_2) \qquad 4.2.14$$

Substitution of this value for the reactive intermediate into the rate law for the formation of products, Eq. 4.2.8, gives:

$$\frac{d[P]}{dt} = \frac{k_2 k_1 [RX] [Nuc{:}^-]}{k_{-1} [X^-] + k_2 [Nuc{:}^-]} \qquad (k_1 << k_2) \qquad 4.2.15$$

We can unravel this rate law by looking at some appropriate limits. When $k_2 << k_{-1}$, the second term in the denominator is negligible and Eq. 4.2.15 gives the same result as the pre-equilibrium mechanism, Eq. 4.2.12. You can think of the pre-equilibrium mechanism as a special case of the steady-state approximation. Now consider the beginning of the reaction when $[X^-]$ is small and $[Nuc{:}^-]$ is large. The first term in the denominator becomes negligible and Eq. 4.2.15 reduces to:

$$\frac{d[P]}{dt} = k_1 [RX] \qquad (k_{-1} [X^-] << k_2 [Nuc{:}^-]) \qquad 4.2.16$$

This rate law is first-order in reactant, which is why the mechanism is called first-order nucleophillic substitution or S_N1 for short. At the beginning, the reaction is also correspondingly zeroth-order in $Nuc{:}^-$. On the other extreme, at the end of the reaction, $[X^-]$ is large and $[Nuc{:}^-]$ is small. The second term in the denominator is now negligible and:

$$\frac{d[P]}{dt} = \frac{k_2 k_1 [RX] [Nuc{:}^-]}{k_{-1} [X^-]} \qquad (k_{-1} [X^-] >> k_2 [Nuc{:}^-]) \qquad 4.2.17$$

In other words, as the reaction proceeds, the reaction approaches first-order in $Nuc{:}^-$. In this limit, which corresponds to a rapid reverse process for the first step, the results follow the pre-equilibrium mechanism; Eqs. 4.2.17 and 4.2.12 are the same. One way to verify S_N1 behavior is to add $[X^-]$ in large excess at the <u>beginning</u> of the reaction. With a large excess of $[X^-]$, the concentration of X^- can be considered constant and can be grouped with the rate constant:

$$\frac{d[P]}{dt} = \left(\frac{k_2 k_1}{k_{-1} [X^-]}\right)_{cst} [RX] [Nuc{:}^-] \qquad (\text{excess } [X^-]) \qquad 4.2.18$$

With a large excess of $[X^-]$, the reaction shifts to second-order overall. The shifts in observed rate law define S_N1 behavior: zeroth-order in $[Nuc{:}^-]$ at the beginning, first-order in $[Nuc{:}^-]$ at the end, and second-order overall with a large excess of $[X^-]$.

At Steady-State All Elementary Steps Have the Same Net Rate: We can use the S_N1 example to make several general comments about multi-step mechanisms. First note that during the vast majority of this S_N1 reaction, the net rate of the reversible first step is equal to the rate of the second step. To show this equivalence, we can rearrange Eq. 4.2.13 to show that:

$$\begin{array}{ccccc} k_1\,[RX] & - & k_{-1}\,[R^+]\,[X^-] & = & k_2\,[R^+]\,[Nuc:^-] \\ \upsilon_+ & - & \upsilon_- & = & \upsilon_2 \\ \textit{forward rate} & & \textit{reverse rate} & & \\ & \textit{first step} & & & \textit{second step} \end{array} \qquad (k_1 << k_2) \qquad 4.2.19$$

where we recognize the terms on the left as the net rate for the reversible first step, Eq. 4.2.9, and the term on the right of the equality as the rate of the unidirectional second step, υ_2. This result is worth thinking about. In general for multi-step mechanisms, during the majority of the course of the reaction the net rates for each step are equal, regardless of which step is the rate determining step.[2] In this specific case, the second step can’t run any faster than the first reversible process. The second step is limited by the supply of the intermediate. On the other hand, if the second step is slower than the first reversible step, then the second step becomes the rate determining step. If the second step is the rate determining step, the net rate of the first reversible process can’t run any faster than the second step at steady state. This is the reason we have used the terminology: “the intrinsically slow step is the rate determining step.” We need to differentiate between rates and rate constants. For first-order reactions, the intrinsic rate of a unidirectional elementary step is determined by the rate constant for the step. For first-order reactions, steps with small rate constants are intrinsically slow. But, during the vast majority of the reaction all the steps have the same net rate, if the steady-state approximation applies.

What about the rate determining step for more complex mechanisms? For second-order processes, the concentration of both reactants affect the rate of the mechanistic steps and a complete analysis of the integrated rate laws for the mechanism is required to determine the rate determining step, if there is one.

Many reactions follow a reversible first-step mechanism. In atmospheric environmental chemistry one important reaction is the destruction of ozone through the disproportionation:

$$2\,O_3 \rightarrow 3\,O_2 \qquad 4.2.20$$

which is postulated to follow the mechanism:

$$O_3 \underset{k_{-1}}{\overset{k_1}{\rightleftarrows}} O_2 + O \qquad\qquad O + O_3 \overset{k_2}{\rightarrow} 2\,O_2 \qquad 4.2.21$$

Another important use of the steady-state approximation is for simplifying the rate laws for enzyme mechanisms.

Reversible First-Step Mechanism — Michaelis-Menten Mechanism: Enzyme catalysis can follow a wide variety of mechanisms. However, many enzyme catalyzed reactions follow the mechanism:

$$E + S \underset{k_{-1}}{\overset{k_2}{\rightleftarrows}} ES \overset{k_1}{\rightarrow} P + E \qquad 4.2.22$$

where E is the enzyme, S is the substrate, and ES is the enzyme-substrate complex. Only the stoichiometry differs from the reversible first-step mechanism that we just discussed. We can use this example to highlight the general principles involved in using the steady-state approximation for any problem. We start by writing the rate law for the formation of product:

$$\upsilon = \frac{d[P]}{dt} = k_1\,[ES] \qquad 4.2.23$$

If the rate law for the production of product involves the concentrations of reactive intermediates, we use the steady-state approximation to solve for the concentrations of the reactive intermediates. Focusing on the rate law for the formation of the enzyme-substrate complex and applying the steady-state approximation gives:

$$\frac{d[ES]}{dt} = k_2\,[E][S] - k_{-1}\,[ES] - k_1\,[ES] \approx 0 \qquad (k_2 << k_1) \qquad 4.2.24$$

Solving for the enzyme-substrate complex concentration gives:

$$[ES] = \frac{k_2\,[E][S]}{(k_{-1} + k_1)} \qquad (k_2 << k_1) \qquad 4.2.25$$

At this point, however, we have two unknown concentrations, [E] and [S]. We can use the mass balance for the total enzyme concentration, $[E]_o = [E] + [ES]$, to eliminate one of the unknowns. Solving the mass balance to give the free enzyme concentration gives:

$$[E] = [E]_o - [ES] \qquad 4.2.26$$

Substitution of this free enzyme concentration into Eq. 4.2.25 results in:

$$[ES] = \frac{k_2\,([E]_o - [ES])[S]}{(k_{-1} + k_1)} = \frac{k_2\,[E]_o[S]}{(k_{-1} + k_1)} - \frac{k_2\,[ES][S]}{(k_{-1} + k_1)} \qquad (k_2 << k_1) \qquad 4.2.27$$

Collecting terms in [ES] gives:

$$[ES]\left(1 + \frac{k_2\,[S]}{(k_{-1} + k_1)}\right) = \frac{k_2\,[E]_o[S]}{(k_{-1} + k_1)} \qquad (k_2 << k_1) \qquad 4.2.28$$

and solving for [ES] gives the steady-state enzyme-substrate complex concentration:

$$[ES] = \frac{k_2\,[E]_o[S]}{(k_{-1} + k_1 + k_2[S])} \qquad (k_2 << k_1) \qquad 4.2.29$$

This last equation can then be substituted back into the rate law for the formation of product, Eq. 4.2.23 to give the final result:

$$\frac{d[P]}{dt} = k_1\,[ES] = k_1\,\frac{k_2\,[E]_o[S]}{(k_{-1} + k_1 + k_2[S])} \qquad (k_2 << k_1) \qquad 4.2.30$$

which is the final desired approximate rate law. This equation is usually rearranged by dividing both the numerator and denominator by k_2 and defining the **Michaelis constant** as:

$$k_M \equiv \frac{(k_1 + k_{-1})}{k_2} \qquad 4.2.31$$

In terms of the Michaelis constant the rate law is:

$$\upsilon = \frac{d[P]}{dt} = k_1\,[ES] = \frac{k_1\,[E]_o[S]}{(k_M + [S])} \qquad (k_2 << k_1) \qquad 4.2.32$$

In homework Problem 13 you will show that the integrated time course for the reaction is linear for short times and then the rate decreases as [P] approaches $[P]_\infty = [S]_o$, Figure 4.2.1.

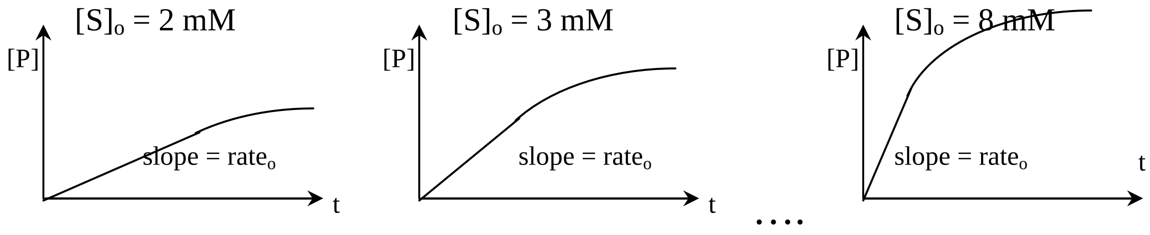


Figure 4.2.1: Enzyme characterization through a Michaelis – Menten kinetic study. Separate initial rate determinations are first run while varying the initial concentration of substrate.

In a typical enzyme kinetics study, the initial rate, υ_o, is determined while varying the initial substrate concentration in a series of experiments. One convenient method for extracting the values of k_1 and k_M is given by a double reciprocal plot, Figure 4.2.2. First, Eq. 4.2.32 is inverted:

$$\frac{1}{\upsilon} = \frac{k_M + [S]}{k_1\,[E]_o[S]} \qquad (k_2 << k_1) \qquad 4.2.33$$

The initial rate, υ_o, corresponds to: $[S] \approx [S]_o$:

$$\frac{1}{\upsilon_o} = \frac{1}{k_1\,[E]_o} + \frac{k_M}{k_1\,[E]_o[S]_o} \qquad (k_2 << k_1, \text{initial rate}) \quad 4.2.34$$

What is the maximum rate for this reaction? The maximum, υ_{max} occurs when all the available enzyme is bound to substrate: $[ES] = [E]_o$. Substituting the maximum enzyme-substrate complex concentration into the rate law, Eq. 4.2.23 gives:

$$\upsilon_{max} = \frac{d[P]}{dt} = k_1\,[E]_o \qquad (\text{maximum rate}) \qquad 4.2.35$$

Substituting the maximum rate into Eq. 4.2.33 gives:

$$\frac{1}{\upsilon_o} = \frac{1}{\upsilon_{max}} + \frac{k_M}{\upsilon_{max}\,[S]_o} \qquad (k_2 << k_1, \text{initial rate}) \quad 4.2.36$$

This form is called a **double-reciprocal** plot because the reciprocal of the initial rate versus the reciprocal of the initial substrate concentration gives a straight line with slope = k_M/υ_{max} and intercept $1/\upsilon_{max}$, Figure 4.2.2. This plot is often called a Lineweaver-Burk plot. However, double-reciprocal plots often produce high correlation coefficients between the fit coefficients and in addition the error propagation in calculating υ_{max} from 1/intercept is quite unfavorable. There are several better ways to extract K_M and k_2, which you will study if you take biochemistry. A non-linear curve fit directly using the functional form from Eq. 4.2.32 often works best. Curve fitting for Eq. 4.2.15 has the same issues and can be handled in double-reciprocal or non-linear forms.

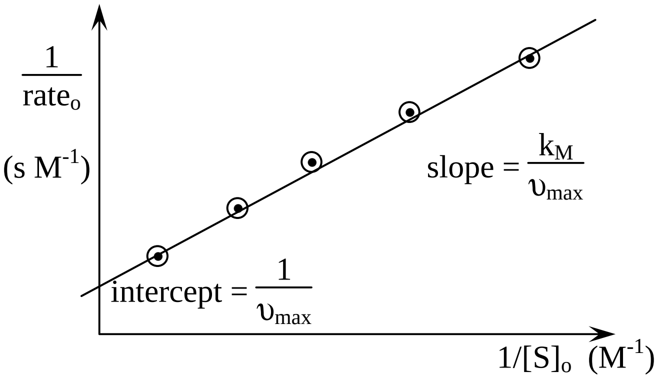


Figure 4.2.2: Lineweaver-Burk plot for the determination of υ_{max} and the Michaelis constant.

The two available parameters from fitting Eq. 4.2.32, υ_{max} and k_M, are not sufficient to determine k_2 or k_{-1} separately. Stopped flow or temperature-jump chemical relaxation experiments are usually necessary to completely determine all the rate constants in the mechanism. Please also note that the Michaelis-Menten mechanism is just one of the simplest possible mechanisms for enzyme catalyzed reactions. In particular, many enzyme reactions show inhibition or allosteric control, which are not taken into account by the Michaelis-Menten mechanism. Our purpose here is to show a useful and interesting example of the steady-state approximation.

To summarize the use of the steady-state approximation, the steps are:

Steady-State Approximation:
1. Write the rate law for the formation of product.
2. Write the rate laws for the formation of any reactive intermediates.
3. Apply the steady-state approximation to all the reactive intermediates to obtain approximate concentrations of the reactive intermediates.
4. Mass balance equations are often applied to decrease the number of unknowns, one fewer unknown for each mass balance.
5. Substitute the reactive intermediate concentrations into the final rate law.
6. Look at appropriate limits to further simplify the rate law and compare to the rate laws for simpler mechanisms. For example, assume short times or very long times or assume some rate constants are much larger than others.
7. Integrate the approximate rate law if desired.

The steady-state approximation cannot be applied if the concentrations of the intermediates are not much less than the initial concentration of reactants. The steady-state approximation should not be applied to reactants or products.

Another important example of the steady-state approximation and the reversible first-step mechanism is the theory of unimolecular reactions. In particular, if elementary mechanistic steps describe the collisions that take place, how can a reaction be first-order? A unimolecular step implies that no collision occurs; how can the reactant overcome its activation energy barrier?

Example 4.2.2: *Unimolecular Reactions – Lindemann-Henshelwood Mechanism*
The following mechanism has been postulated to explain first-order reactions.

$$A + A \underset{k_{-2}}{\overset{k_2}{\rightleftarrows}} A + A^* \qquad\qquad A^* \xrightarrow{k_1} B + C \qquad 4.2.37$$

where A* is an activated molecule created by the collision. Use the steady-state approximation to show that this mechanism results in a first-order rate law under suitable circumstances (i.e., apply step 6 in the general scheme, above).

Answer: Step 1: The rate law for the formation of products is:

$$\frac{d[B]}{dt} = k_1[A^*] \qquad 4.2.38$$

Step 2: The rate law for the formation of the activated molecule as the reactive intermediate is:

$$\frac{d[A^*]}{dt} = k_2[A]^2 - k_{-2}\,[A^*][A] - k_1[A^*] = 0 \qquad (k_2 << k_1) \qquad 4.2.39$$

Step 3: Setting this last equation equal to zero to apply the steady-state approximation and solving for the reactive intermediate gives:

$$[A^*] = \frac{k_2[A]^2}{k_{-2}[A] + k_1} \qquad (k_2 << k_1) \qquad 4.2.40$$

Step 4: There is only one unknown in this last equation, so we can skip step 4.
Step 5: Substitution of the concentration of the reactive intermediate into the rate law for the formation of products, Eq. 4.2.38, gives the final simplified rate law:

$$\frac{d[B]}{dt} = k_1[A^*] = \frac{k_1k_2[A]^2}{k_{-2}[A] + k_1} \qquad (k_2 << k_1) \qquad 4.2.41$$

Step 6: The limit $k_1 << k_{-2}[A]$ corresponds to rapid deactivation of the excited molecule. The second term in the denominator of this last equation is negligible and:

$$\frac{d[B]}{dt} = \frac{k_1k_2}{k_{-2}}\,[A] \qquad (k_2 << k_1,\ k_1 << k_{-2}[A]) \qquad 4.2.42$$

The net result is an overall first-order reaction, as we wished to find.

Note, however, that when $k_{-2}[A] << k_1$ the reverse process in the reversible first step is slow. This limit is called slow deactivation, which from Eq. 4.2.41 leads to:

$$\frac{d[B]}{dt} = k_2[A]^2 \qquad (k_2 << k_1,\ k_{-2}[A] << k_1) \qquad 4.2.43$$

which is an overall second-order process. The validity of the Lindemann-Henshelwood mechanism, in an analogous way to the S_N1 mechanism, is often verified by determining the rate law under different experimental conditions. At low initial pressure, A + A* collisions are unlikely, $k_{-2}[A] << k_1$, so that the deactivation step is slow giving a second-order reaction. At high pressure, A + A* collisions are likely so deactivation is fast. At high pressure, $k_1 << k_{-2}[A]$, and the unimolecular step is the intrinsically slow step giving a first-order process.

As you review these last three reversible first-step mechanisms, make sure to note the similarities in the final rate laws, Eqs. 4.2.15, 4.2.30, and 4.2.41. This general mechanism is one example of how a particular mechanistic sequence can often be used in a variety of different applications. Continuing with our theme of exploring the steady-state approximation, we now wish to consider more complex multi-step reaction mechanisms. Again, there are many possible application areas to choose from to illustrate the underlying principles. Many atmospheric, aquatic, and enzymatic reactions involve reactive intermediates that participate in a series of steps. These reactions often follow chain mechanisms. The processes that are involved in stratospheric ozone destruction and tropospheric ozone production are chain mechanisms.

4.3 Chain Mechanisms and Autocatalysis

Chain mechanisms are characterized by steps that consume a reactive intermediate but then produce one or more reactive intermediates. Such steps are called **chain propagation** steps, because the net number of reactive species is kept constant or increased. The reactive intermediates in chain propagation steps are called **chain carriers**. Odd electron atoms and molecules are often called free radicals. Chain carriers are often free radicals. For example, chlorine and bromine atoms, Cl• and Br•, are reactive odd-electron atoms that are chain carriers in stratospheric ozone depletion.

The gas phase reaction:

$$H_2 + Br_2 \rightarrow 2\,HBr \qquad 4.3.1$$

is a well-studied chain reaction. The experimentally determined rate law is quite complex:

$$\frac{d[HBr]}{dt} = \frac{k_a\,[H_2][Br_2]^{1/2}}{k_b + k_c\dfrac{[HBr]}{[Br_2]}} \qquad 4.3.2$$

where k_a, k_b, and k_c are empirically determined rate constants. The goal is to postulate a mechanism that agrees with the experimental rate law. The proposed mechanism is:

$$Br_2 \xrightarrow{k_1} 2\,Br \qquad \text{initiation} \qquad 4.3.3$$

$$Br + H_2 \xrightarrow{k_2} HBr + H \qquad \text{propagation} \qquad 4.3.4$$

$$H + Br_2 \xrightarrow{k_3} HBr + Br \qquad \text{propagation} \qquad 4.3.5$$

$$H + HBr \xrightarrow{k_4} H_2 + Br \qquad \text{inhibition} \qquad 4.3.6$$

$$2\,Br \xrightarrow{k_5} Br_2 \qquad \text{breaking} \qquad 4.3.7$$

A chain mechanism begins with the formation of a chain carrier in an **initiation** step, which is Eq. 4.3.3 for this reaction. The chain initiation step is followed by one or more chain propagation steps. The propagation steps cycle producing products and additional reactive intermediates that carry the chain mechanism forward. The chain propagation steps compete with reactions that decrease the net number of chain carriers. Such steps are called **chain breaking** steps. Chain reactions are often quite rapid because the chain propagation steps don’t consume the net pool of reactive intermediates. Chain breaking steps slow the overall process. On the other hand, some steps consume product, but still produce chain carriers. Such steps are called **chain inhibition** steps.

It is not obvious that the above proposed mechanism results in the observed rate law. One approach is to use the steady-state approximation to simplify the overall rate law. In this mechanism, the free radical chain carriers are H and Br atoms. We need to apply the steady-state approximation to both of these reactive intermediates. Following step 1 in the general steady-state approximation scheme, we formulate the rate law for the formation of products:

$$\frac{d[HBr]}{dt} = k_2\,[Br][H_2] + k_3\,[H][Br_2] - k_4\,[H][HBr] \qquad 4.3.8$$

This rate law is clearly not in the final desired form because the free radical chain carriers are involved. This juncture is where we can use the steady-state approximation to solve for the steady-state concentrations of the reactive intermediates. Writing the rate laws for [Br] and [H] gives:

$$\frac{d[Br]}{dt} = 2\,k_1[Br_2] - k_2\,[Br][H_2] + k_3\,[H][Br_2] + k_4\,[H][HBr] - 2\,k_5[Br]^2 = 0 \qquad 4.3.9$$

$$\frac{d[H]}{dt} = k_2\,[Br][H_2] - k_3\,[H][Br_2] - k_4\,[H][HBr] = 0 \qquad 4.3.10$$

where we set each rate law equal to zero to satisfy the steady-state approximation. These last two equations provide two simultaneous equations in two unknowns, [H] and [Br]. If we add Eq. 4.3.9 and 4.3.10, the middle terms of Eq. 4.3.9 cancel and the result is:

$$0 = 2\,k_1[Br_2] - 2\,k_5\,[Br]^2 \qquad 4.3.11$$

which we can solve for the Br atom concentration:

$$[Br] = \left(\frac{k_1}{k_5}\,[Br_2]\right)^{½} \qquad 4.3.12$$

We can solve Eq. 4.3.10 for the hydrogen atom concentration and then substitute for [Br] from this last equation:

$$[H] = \frac{k_2[Br][H_2]}{k_3[Br_2] + k_4[HBr]} = \frac{k_2\left(\frac{k_1}{k_5}\right)^{½}[H_2][Br_2]^{½}}{k_3[Br_2] + k_4[HBr]} \qquad 4.3.13$$

Notice that Eq. 4.3.8 and 4.3.10 have the $k_2[Br][H_2]$ and $-\,k_4[H][HBr]$ terms in common. Since Eq. 4.3.10 is equal to zero, subtracting Eq. 4.3.10 from Eq. 4.3.8 will cancel the common terms, but leave the overall rate law unchanged:

$$\begin{array}{rl} \frac{d[HBr]}{dt} & = k_2\,[Br][H_2] + k_3\,[H][Br_2] - k_4\,[H][HBr] \\ -(\quad 0 & = k_2\,[Br][H_2] - k_3\,[H][Br_2] - k_4\,[H][HBr]\ ) \\ \hline \frac{d[HBr]}{dt} & = \qquad 2\,k_3\,[H][Br_2] \end{array} \qquad 4.3.14$$

Finally, we can substitute the H atom concentration from Eq. 4.3.13 into Eq. 4.3.14 to give:

$$\frac{d[HBr]}{dt} = \frac{2\ k_3\ k_2\ k_1^{1/2}\ k_5^{-1/2}\ [H_2][Br_2]^{1/2}\ [Br_2]}{k_3[Br_2] + k_4[HBr]} \qquad 4.3.15$$

Dividing numerator and denominator of this last equation by $[Br_2]$ gives the final result:

$$\frac{d[HBr]}{dt} = \frac{2\ k_3\ k_2\ k_1^{1/2}\ k_5^{-1/2}\ [H_2][Br_2]^{1/2}}{k_3 + k_4\dfrac{[HBr]}{[Br_2]}} \qquad 4.3.16$$

which is an equation only a chemical kineticist could love. Comparison of this final result with the experimentally determined rate law, Eq. 4.3.2, shows the same functional form. The experimental rate constant k_a turns out to be a composite of the rate constants for several mechanistic steps, while $k_b = k_3$, and $k_c = k_4$. It is important to note that the agreement between the experimentally determined rate law and the rate law predicted by the proposed mechanism does not prove that the mechanism is the correct mechanism, nor the only mechanism.

Notice also that this proposed mechanism has two reversible steps: Eqs. 4.3.3 and 4.3.7 and also Eqs. 4.3.4 and 4.3.6. The full mechanism can be equivalently written as:

$$Br_2 \underset{k_5}{\overset{k_1}{\rightleftarrows}} 2\ Br \qquad \text{initiation, breaking} \qquad 4.3.17$$

$$Br + H_2 \underset{k_4}{\overset{k_2}{\rightleftarrows}} HBr + H \qquad \text{propagation} \qquad 4.3.18$$

$$H + Br_2 \xrightarrow{k_3} HBr + Br \qquad \text{propagation} \qquad (4.3.5)\ 4.3.19$$

The reverse of Eq. 4.3.5 (4.3.19) was not considered. Because all the proposed steps are not reversible, this proposed mechanism violates a principle called “detailed balance,” which we will discuss in Sec. 4.5. As a consequence, this mechanism can only hold at the beginning of the reaction, away from equilibrium. As the reaction approaches equilibrium the proposed complete mechanism must include the exact reverse of each mechanistic step. However, this example serves nicely to introduce the concept of chain initiation, propagation, and termination steps. The trick we used in adding and subtracting rate laws, in the derivation of Eqs. 4.3.11 and 4.3.14, is a very handy technique to master as you work further on complex mechanisms. However, there is one more important type of mechanistic step for chain mechanisms.

What causes explosions? Explosions are very rapid and exothermic chemical reactions that have chain branching steps. A **chain branching** step is a chain propagation step that increases the net number of chain carriers. The reaction $2\ H_2 + O_2 \rightarrow 2\ H_2O$ is a very important example:

$$\begin{array}{lll}
H_2 + O_2 \rightarrow 2\ {\bullet}OH & \text{initiation} & \\
H_2 + {\bullet}OH \rightarrow H{\bullet} + H_2O & \text{propagation} & \\
O_2 + H{\bullet} \rightarrow {\bullet}O{\bullet} + {\bullet}OH & \text{branching} & \\
{\bullet}O{\bullet} + H_2 \rightarrow {\bullet}OH + H{\bullet} & \text{branching} & 4.3.20
\end{array}$$

The odd electron, neutral hydroxyl radical, $\bullet OH$, is also an important free radical intermediate in many atmospheric process. Remember that ground state oxygen molecules have two unpaired electrons; the electron configuration for ground state oxygen is $2s^2\ 2p^4$:

$$\text{O :}\ \underset{2s}{\boxed{\uparrow\downarrow}} \qquad \underset{2p}{\boxed{\uparrow\downarrow}\boxed{\uparrow}\boxed{\uparrow}} \qquad \text{(ground state)} \tag{4.3.21}$$

Such an electron configuration results in a **triplet state**. For convenience we symbolize triplet oxygen atoms in the mechanism as •O•. Triplet oxygen atoms are highly reactive. (Remember also that ground state oxygen molecules also have two unpaired electrons and can be correspondingly symbolized as $\bullet O_2\bullet$.) Notice that the two chain branching steps consume one reactive intermediate on the left and produce two reactive intermediates on the right. The net increase in chain carriers in chain branching steps greatly accelerates the reaction to the point of an explosion. The rapid and highly exothermic nature of the oxidation of hydrogen makes hydrogen a useful fuel. Hydrogen was the fuel for the main engines of the space shuttle. Many feel that hydrogen should become a commonly used transportation fuel as a substitute for petroleum. Chain branching steps are a specific example of a process called autocatalysis, which has important applications in some enzymatic reactions as well as combustion reactions.

In Autocatalysis, Products Catalyze the Reaction: Autocatalysis is a more general phenomenon than chain branching. A simple form of autocatalysis is given by the following reaction with a rate law that includes the concentration of the product:

$$A \xrightarrow{k_{AB}} B \qquad\qquad -\frac{d[A]}{dt} = k_{AB}\,[A][B] \tag{4.3.22}$$

where a corresponding single-step mechanism might be (among other possibilities):

$$A + B \rightarrow 2\,B \tag{4.3.23}$$

The rate of an autocatalytic process increases with the production of products, because a product is also a reactant for one or more mechanistic steps. An autocatalytic reaction requires that both A and B be present at the beginning of the reaction, $[B]_o \neq 0$, otherwise Eq. 4.3.23 cannot occur. Autocatalytic processes show an induction period, followed by a rapid rise in rate as products begin to build, but then finally the rate slows as a reactant is exhausted. A simple first-order reaction is compared to an autocatalytic process for equal rate constants in Figure 4.3.1.

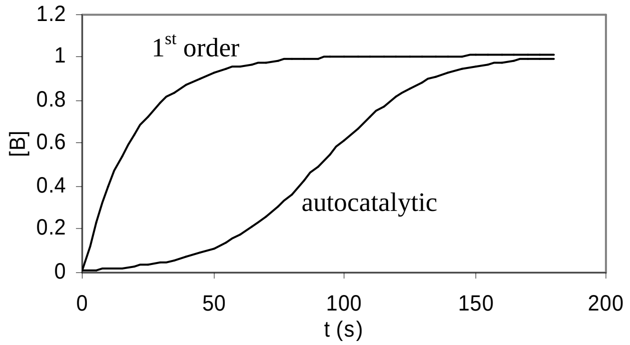


Figure 4.3.1: An autocatalytic process shows an induction period, followed by a rapid increase in rate. The rate constant is $0.1\ s^{-1}$ and $[A]_o = 1$ M, $[B]_o = 0.01$ M. The initial rate of the autocatalytic process is $\upsilon_o = 0.1\ s^{-1}$ (1 M)(0.01 M), as compared to a corresponding first-order reaction with $\upsilon_o = 0.1\ s^{-1}$ (1 M), giving the slow initial rate for the autocatalytic process.

Autocatalysis is an example of a general phenomenon called positive feedback. The loud squeal sometimes heard from public address systems is another commonly encountered example of positive feedback. A biochemical example of an autocatalytic reaction is the conversion of trypsinogen to trypsin. Trypsin is a proteolytic enzyme used to degrade proteins in food in the digestive system. The pancreas maintains a pool of inactive trypsinogen until required, at which point a small peptide is cleaved from the terminus of trypsinogen to produce active trypsin in an autocatalytic process. The autocatalysis provides the rapid conversion of the enzyme to meet the sudden demand of the digestive system. Other examples include the combustion of hydrocarbons at high temperatures[3] and the degradation of aspirin to salicylic acid and acetic acid. Autocatalytic steps also play an important role in some clock reactions and especially oscillating reactions, which operate far from equilibrium.

Example 4.3.1: *Chain Mechanisms and Autocatalysis*
Iron is a necessary nutrient for bacterial and phytoplankton growth in natural waters. The biological availability of iron is dependent on the relative amounts, that is the speciation, of iron between Fe(II) and Fe(III). Fe(II) is readily soluble and easily acquired by bacteria and phytoplankton. However, Fe(III) forms hydroxo complexes and $Fe(OH)_3$ that precipitates iron from solution and makes Fe(III) unavailable. In oxygenated waters, Fe(II) is rapidly oxidized to Fe(III), making the pool of iron largely unavailable for biological growth. The mechanism for Fe(II) oxidation has been extensively studied because of its importance for regulating biological productivity. The proposed mechanism is:[4]

$$\mathrm{Fe(II)} + \mathrm{O_2} \xrightarrow{k_1} \mathrm{Fe(III)} + \bullet\mathrm{O_2^-} \tag*{4.3.24}$$

$$\mathrm{Fe(II)} + \bullet\mathrm{O_2^-} + 2\,\mathrm{H^+} \xrightarrow{k_2} \mathrm{Fe(III)} + \mathrm{H_2O_2} \tag*{4.3.25}$$

$$\mathrm{Fe(II)} + \mathrm{H_2O_2} \xrightarrow{k_3} \mathrm{Fe(III)} + \bullet\mathrm{OH} + \mathrm{OH^-} \tag*{4.3.26}$$

$$\mathrm{Fe(II)} + \bullet\mathrm{OH} \xrightarrow{k_4} \mathrm{Fe(III)} + \mathrm{OH^-} \tag*{4.3.27}$$

The $\bullet O_2^-$ ion is called superoxide. Since the reaction is run at constant pH, the rate constant for Eq. 4.3.25 is defined to include the $[H^+]$ concentration, $k_2 = k_2'[H^+]^2$. The pool of reactive oxygen intermediates, $\bullet O_2^-$, H_2O_2, and $\bullet OH$, also plays an important role in many other aquatic redox processes. (a) Find the overall stoichiometry. (b) Identify the chain initiation, propagation, termination steps. (c) Is this process autocatalytic? (d) Use the steady-state approximation to find the steady-state concentrations of the reactive intermediates. (e) Find the overall rate law under steady-state conditions.

Answer: (a) Adding all four steps gives a 4:1 stoichiometry for Fe(II) oxidation by O_2:

$$4\,\mathrm{Fe(II)} + \mathrm{O_2} + 2\,\mathrm{H^+} \rightarrow 4\,\mathrm{Fe(III)} + 2\,\mathrm{OH^-} \tag*{4.3.28}$$

(b) Step 1 is the chain initiation step. Steps 2 and 3 are chain propagation steps. Step 4 is a chain termination step, since no reactive oxygen species is produced.
(c) This mechanism is autocatalytic, especially at high Fe(II) concentrations, since the products of reactions 1, 2, and 3 further react with reactant Fe(II).[4]
(d) The rate laws are:

$$\frac{d[\mathrm{Fe(II)}]}{dt} = -k_1 [\mathrm{Fe(II)}][\mathrm{O_2}] - k_2 [\mathrm{Fe(II)}][\bullet\mathrm{O_2^-}] - k_3 [\mathrm{Fe(II)}][\mathrm{H_2O_2}] - k_4 [\mathrm{Fe(II)}][\bullet\mathrm{OH}] \qquad 4.3.29$$

$$\frac{d[\bullet\mathrm{O_2^-}]}{dt} = k_1 [\mathrm{Fe(II)}][\mathrm{O_2}] - k_2 [\mathrm{Fe(II)}][\bullet\mathrm{O_2^-}] = 0 \qquad 4.3.30$$

$$\frac{d[\mathrm{H_2O_2}]}{dt} = k_2 [\mathrm{Fe(II)}][\bullet\mathrm{O_2^-}] - k_3 [\mathrm{Fe(II)}][\mathrm{H_2O_2}] = 0 \qquad 4.3.31$$

$$\frac{d[\bullet\mathrm{OH}]}{dt} = k_3 [\mathrm{Fe(II)}][\mathrm{H_2O_2}] - k_4 [\mathrm{Fe(II)}][\bullet\mathrm{OH}] = 0 \qquad 4.3.32$$

Applying the steady-state approximation for $\bullet\mathrm{O_2^-}$ from Eq. 4.3.30 gives:

$$k_1 [\mathrm{Fe(II)}][\mathrm{O_2}] = k_2 [\mathrm{Fe(II)}][\bullet\mathrm{O_2^-}] \quad \text{or} \quad [\bullet\mathrm{O_2^-}] = \frac{k_1 [\mathrm{O_2}]}{k_2} \qquad 4.3.33$$

Applying the steady-state approximation for H_2O_2 from Eq. 4.3.31 and substitution of Eq. 4.3.33 for $[\bullet\mathrm{O_2^-}]$ gives:

$$k_2 [\mathrm{Fe(II)}][\bullet\mathrm{O_2^-}] = k_3 [\mathrm{Fe(II)}][\mathrm{H_2O_2}] \quad \text{or} \quad [\mathrm{H_2O_2}] = \frac{k_2 [\bullet\mathrm{O_2^-}]}{k_3} = \frac{k_1 [\mathrm{O_2}]}{k_3} \qquad 4.3.34$$

Similarly the steady-state approximation for •OH with Eq. 4.3.34 gives:

$$k_3 [\mathrm{Fe(II)}][\mathrm{H_2O_2}] = k_4 [\mathrm{Fe(II)}][\bullet\mathrm{OH}] \quad \text{or} \quad [\bullet\mathrm{OH}] = \frac{k_3 [\mathrm{H_2O_2}]}{k_4} = \frac{k_1 [\mathrm{O_2}]}{k_4} \qquad 4.3.35$$

The concentrations of the reactive intermediates at steady state are then predicted to be completely determined from the dissolved O_2 concentration. The ratios of the reactive intermediates are completely fixed at steady state by k_2, k_3, and k_4.

(e) To find the overall rate law at steady state, we first need to look for a simplification of Eq. 4.3.29. Notice from Eqs. 4.3.33-4.3.35 that:

$$k_1 [\mathrm{Fe(II)}][\mathrm{O_2}] = k_2 [\mathrm{Fe(II)}][\bullet\mathrm{O_2^-}] = k_3 [\mathrm{Fe(II)}][\mathrm{H_2O_2}] = k_4 [\mathrm{Fe(II)}][\bullet\mathrm{OH}] \qquad 4.3.36$$

Then, substitution of k_1 [Fe(II)][O_2] for each of the last three terms in Eq. 4.3.29 gives:

$$\frac{d[\mathrm{Fe(II)}]}{dt} = -4\, k_1 [\mathrm{Fe(II)}][\mathrm{O_2}] \qquad 4.3.37$$

In natural waters, the saturated [O_2] concentration at 20°C is near 230-284 μM and the total iron concentration is often in the micromolar or sub-micromolar range. In well-oxygenated water, then, O_2 is often in excess and the oxidation of Fe(II) is a pseudo-first order process.

4.4 Oscillating Reactions

Oscillating Chemical Reactions are Autocatalyic Reactions Far from Equilibrium: Reactions near equilibrium approach equilibrium with simple exponential time dependence, Sec. 3.6. Classical kinetics leads us to expect smooth monotonic time evolution towards the equilibrium state. It is quite striking then to discover that some reactions show concentration profiles that oscillate in time. The Briggs-Rauscher reaction is one of the first laboratory examples of an oscillating reaction, Figure 4.4.1.[5] The mechanism of the reaction is complex, but the bulk of the reaction is the oxidation of malonic acid by iodate and hydrogen peroxide in acidic solution:

$$IO_3^- + 2\ H_2O_2 + CH_2(COOH)_2 + H^+ \rightarrow ICH(COOH)_2 + 2\ O_2 + 3\ H_2O \qquad 4.4.1$$

The iodination step is given by:

$$I_2 + CH_2(COOH)_2 \rightarrow ICH(COOH)_2 + I^- + H^+ \qquad 4.4.2$$

where the I_2 is generated and consumed by the reaction of H_2O_2 with IO_3^-:

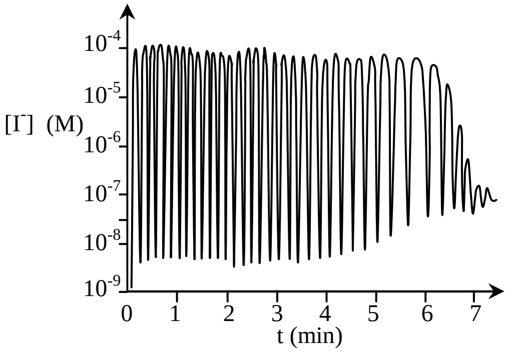
$$5\ H_2O_2 + 2\ IO_3^- + 2\ H^+ \rightarrow I_2 + 5\ O_2\ (g) + 6\ H_2O \qquad 4.4.3$$

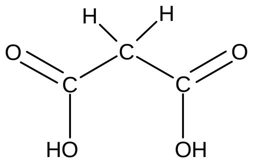
$$5\ H_2O_2 + I_2 \rightarrow 2\ IO_3^- + 2\ H^+ + 4\ H_2O \qquad 4.4.4$$

Adding these two reactions shows that the net result is the catalytic diproportionation of H_2O_2:

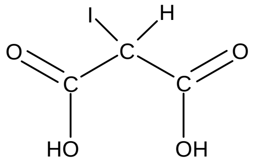
$$2\ H_2O_2 \rightarrow O_2\ (g) + 2\ H_2O \qquad 4.4.5$$

The release of O_2 from the solution and consumption of malonic acid eventually drive the process to equilibrium.





Malonic acid, MA



Iodomalonic acid, IMA

Figure 4.4.1: Periodic variation of I^- during the Briggs-Rauscher reaction, measured by a AgI/Ag electrode.[5] Typical conditions are: $[IO_3^-] = 0.067$ M, $[H_2O_2] = 1.2$ M, $[CH_2(COOH)_2] = 0.050$ M, $[Mn^{2+}] = 0.0067$ M, $[H_2SO_4] = 0.053$ M.

The reaction is catalyzed by Mn^{2+}. Starch indicator is added to detect I_2; the starch-I_2 complex is dark blue. During the reaction, the color changes repeatedly from pale yellow to dark blue and back again. Oscillations continue up to 10 minutes, and then the reaction goes to equilibrium giving a dark blue-black color. The discovery of oscillating reactions was quite unexpected. However, upon reflection, we are surrounded by oscillating reactions. Many oscillating reactions occur in nature that have diurnal, lunar, or seasonal cycles. Alpha and beta brain waves and heart beats are oscillating reactions on shorter time scales. These biological examples show that coupled enzymatic processes can give oscillations, under the proper conditions.

In general, oscillations are common in coupled systems with positive feedback, such as the squeal from public address systems. Positive feedback in chemical systems is provided by an auto-catalytic step. The requirements for an oscillating chemical reaction are:[6]

- The reaction starts far from equilibrium
- The mechanism includes an autocatalytic step, giving positive feedback

Oscillating reactions in closed systems are difficult to construct. However, running oscillating reactions in a flow reactor allows the overall displacement of the system to remain far from equilibrium. A simple mechanism in a flowing system that satisfies the requirements for oscillation is the Lotka-Voltera mechanism:

$$\begin{aligned} M + A &\xrightarrow{k_1} 2\,A \\ A + B &\xrightarrow{k_2} 2\,B \\ B &\xrightarrow{k_3} P \end{aligned} \qquad 4.4.6$$

The M species is supplied at constant concentration. The final product P flows out of the reactor. The first and second step are autocatalytic steps with the rate laws $\upsilon_1 = k_1\,[M]\,[A]$ and $\upsilon_2 = k_2\,[A]\,[B]$. Consider an initial state with high A and low B concentrations. As autocatalytic reaction 2 progresses, the concentration of B increases, gradually at first and then rapidly as in Figure 4.3.1. As the concentration of B increases, A is consumed. The decrease in concentration of A decreases the rate of reaction 1. At that point the concentration of B is high and A is low. Since A is low, the rate of reaction 2 then decreases and B is removed from the reactor by the formation of final product. Then as reaction 1 progresses, the concentration of A increases, gradually at first then rapidly by the autocatalytic process and the cycle repeats.

Example 4.4.1: *Lotka-Volterra Mechanism*
Find the steady state concentrations for the Lotka-Volterra mechanism. Do a numerical simulation with $[M] = 1.0$ M, $[A]_o = 1$ M, $[B]_o = 1$ M, $k_1 = 0.1\ M^{-1}\ s^{-1}$, $k_2 = 0.1\ M^{-1}\ s^{-1}$, $k_3 = 0.05\ s^{-1}$. Then repeat the simulation using the steady state concentrations of A and B.

Answer: The rate laws are:

$$\frac{d[A]}{dt} = -\,k_1[M][A] + 2\,k_1\,[M][A] - k_2\,[A][B] = 0$$

$$\frac{d[B]}{dt} = -\,k_2[A][B] + 2\,k_2\,[A][B] - k_3\,[B] = 0$$

At steady state the rates are zero. Simplifying the two rate laws and factoring out the common concentration gives, respectively:

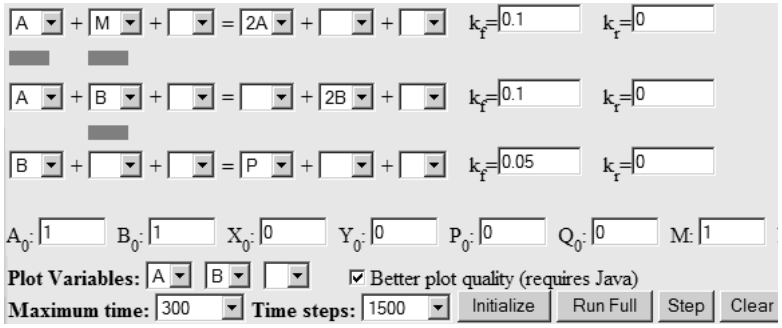
$$[A](k_1\,[M] - k_2\,[B]) = 0 \quad \text{giving} \quad (k_1\,[M] - k_2\,[B]_{ss}) = 0 \quad \text{and then} \quad [B]_{ss} = k_1\,[M]/k_2$$

$$[B](k_2\,[A] - k_3) = 0 \quad \text{giving} \quad (k_2\,[A]_{ss} - k_3) = 0 \quad \text{and then} \quad [A]_{ss} = k_3/k_2$$

For the given conditions: $[B]_{ss} = k_1\,[M]/k_2 = 0.1\ M^{-1}\ s^{-1}(1.0\ M)/\ 0.1\ M^{-1}\ s^{-1} = 1.0\ M$

$[A]_{ss} = k_3/k_2 = 0.05\ s^{-1}/0.1\ M^{-1}\ s^{-1} = 0.5\ M$

www A simulation was run with the "Kinetics Mechanism Simulation" applet on the textbook Web site and on the companion CD, as shown below. The concentrations of A and B oscillate in time, Figure 4.4.2a. A plot of [B] versus [A] shows the relative relationships of the two concentrations. Figure 4.4.2b. After an initial adjustment, the reaction settles into a repeated pattern with the concentration of A is at its maximum when B is near average and the concentration of [B] at maximum when [A] is near average. This closed, repeating time course is called a **cycle**. The simulation with $[A]_o = 0.5$ and $[B]_o = 1$ M corresponds to the steady-state, giving negligible variation in concentrations with time. The center of the cycle corresponds to the steady state concentrations.



A + M + = 2A + + k_f=0.1 k_r=0

A + B + = + 2B + k_f=0.1 k_r=0

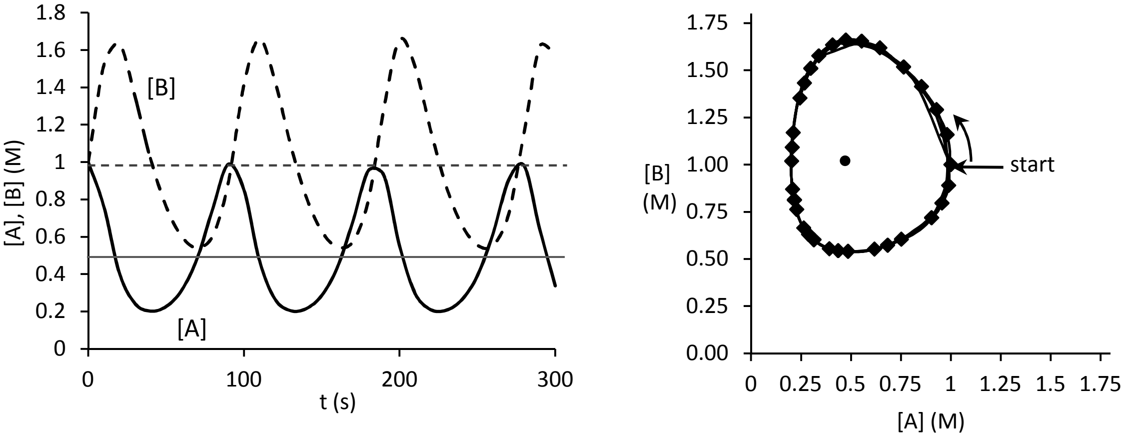
B + + = P + + k_f=0.05 k_r=0

A_0: 1 B_0: 1 X_0: 0 Y_0: 0 P_0: 0 Q_0: 0 M: 1

Plot Variables: A B

☑ Better plot quality (requires Java)

Maximum time: 300 **Time steps:** 1500



(a). (b).

Figure 4.4.2: (a). The Lotka-Volterra mechanism gives oscillatory concentrations for A and B. (b). A plot of [B] versus [A] shows the relative relationships of the concentrations. The concentration of A is at its maximum when B is at its average.

Autocatalysis is the key that allows the system to avoid the monotonic approach to equilibrium. Returning to the Briggs-Rauscher reaction, the postulated mechanism is:[7]

$2\,H^+ + I^- + IO_3^- \rightarrow HOI + HIO_2$	Step 1
$H^+ + HIO_2 + I^- \rightarrow 2\,HOI$	Step 2
$HOI + I^- + H^+ \rightarrow I_2 + H_2O$	Step 3
$HIO_2 + IO_3^- + H^+ \rightarrow 2\,IO_2 + H_2O$	Step 4
$2\,HIO_2 \rightarrow HOI + IO_3^- + H^+$	Step 5
$IO_2 + Mn^{2+} + H_2O \rightarrow HIO_2 + MnOH^{2+}$	Step 6
$H_2O_2 + MnOH^{2+} \rightarrow HO_2 + Mn^{2+} + H_2O$	Step 7
$2\,HO_2 \rightarrow H_2O_2 + O_2$	Step 8
$I_2 + MA \rightarrow IMA + I^- + H^+$	Step 9
$HOI + H_2O_2 \rightarrow I^- + O_2 + H^+ + H_2O$	Step 10

Steps 1-3 consume I^- to produce I_2. Steps 4 and 6 combine to give auto-catalysis. Step 6 shows the Mn^{2+} catalysis of the decomposition of IO_2. Step 7 regenerates the catalyst. Step 8 is a chain termination step. The iodination of malonic acid in step 9 and the reaction of HOI with H_2O_2 generate I^-. Many oscillating inorganic reactions have been designed and oscillating enzymatic reactions are a fertile area of study in biochemistry, systems biology, and ecology.

The Briggs-Rauscher mechanism given above is incomplete, because reverse reactions are not included. The mechanism is only valid far from equilibrium. We now continue with a discussion of how to develop complete mechanisms that hold over the entire course of a chemical reaction and that are consistent with fundamental principles.

4.5 There Are Important Restrictions on Complete Mechanisms

All chemical reactions are inherently reversible at equilibrium. As a reaction proceeds, the system eventually approaches equilibrium when the forward and reverse reaction rates are equal. How is reversibility established for multi-step mechanisms? For a reaction at equilibrium, the forward and reverse reaction rates for each elementary step in the mechanism must be equal. This requirement is called **detailed balance**. For example, consider the possibility of a cyclic reaction with only unidirectional steps:

$$\begin{tikzcd}
& B \arrow[dr, "k_{BC}"] & \\
A \arrow[ur, "k_{AB}"] & & C \arrow[ll, "k_{CA}"']
\end{tikzcd} \tag{4.5.1}$$

The equilibrium state is a time-invariant state. This system may be kept in a time-invariant meta-stable state by a careful balancing of the three steps. However, the system cannot be at equilibrium and equilibrium thermodynamics does not apply.[8] For example, the three unidirectional reactions approach a steady state very differently than a reversible process approaches equilibrium; the approach of the unidirectional steps to a steady state shows oscillatory behavior, but a reversible system approaches equilibrium in an exponential process (e.g., after a temperature or concentration jump, see Problem 34 and Sec. 3.6).[8,9] A given incomplete mechanism might be valid for early stages of a reaction. However for the system to be at equilibrium, each reaction and its exact reverse must occur at the same rate:

$$\begin{tikzcd}
& B \arrow[dl, shift left, "k_{BA}"] \arrow[dr, shift left, "k_{BC}"] & \\
A \arrow[ur, shift left, "k_{AB}"] \arrow[rr, shift left, "k_{AC}"] & & C \arrow[ul, shift left, "k_{CB}"] \arrow[ll, shift left, "k_{CA}"]
\end{tikzcd} \tag{4.5.2}$$

A complete mechanism then includes a series of forward steps and the exact reverse for each step. The sum of the forward processes must give the overall reaction stoichiometry. We now explore the ramifications of the principle of detailed balance to highlight the interrelationships between chemical kinetics and chemical equilibrium. In Chapter 1 we pointed out that chemical kinetics and chemical equilibria are disjoint. The two approaches study very different aspects of chemical reactivity, and it is important not to confuse the two. However, as a reaction approaches equilibrium, chemical kinetics and equilibrium principles must be consistent. This self-consistency is a powerful tool that helps us understand the underlying relationships in reaction mechanisms.

At Equilibrium the Forward and Reverse Reaction Rates are Equal: In Section 3.4 we discussed that at equilibrium the forward and reverse reaction rates are equal. However, in our treatment we needed to assume a specific reaction order for each reactant and product. The relationship between the overall forward and reverse reaction rates must be independent of the molecularity of intervening mechanistic steps. We shouldn't need to make any assumptions in deriving the relationship between the overall forward and reverse rates at equilibrium. Consider an overall reaction, $A + B \rightleftarrows C + D$, composed of three mechanistic steps. For illustration purposes, we choose the steps so that we can illustrate different stoichiometries and a variety of reaction types:

$$2\,A + M \underset{k_{MA}}{\overset{k_{AM}}{\rightleftarrows}} A_2 + M \qquad \text{Step 1} \qquad 4.5.3$$

$$A_2 + B \underset{k_{XA}}{\overset{k_{AB}}{\rightleftarrows}} A + X \qquad \text{Step 2} \qquad 4.5.4$$

$$X \underset{k_{DC}}{\overset{k_{X}}{\rightleftarrows}} C + D \qquad \text{Step 3} \qquad 4.5.5$$

where M is a catalyst or third body and X is a reactive intermediate. The equilibrium constant for this overall reaction is simply:

$$K_{eq} = \left(\frac{[C][D]}{[A][B]}\right)_{eq} \qquad 4.5.6$$

How does this simple equilibrium expression result from the proposed mechanism? According to detailed balance, the forward and reverse rates for each mechanistic step are equal at equilibrium. We can then write equilibrium expressions for each individual mechanistic step:

$$\frac{1}{V}\frac{d\xi_1}{dt} = k_{AM}\,[A]^2[M] - k_{MA}\,[A_2][M] = 0 \qquad K_{eq,1} = \left(\frac{[A_2][M]}{[A]^2[M]}\right)_{eq} \equiv \frac{k_{AM}}{k_{MA}} \qquad 4.5.7$$

$$\frac{1}{V}\frac{d\xi_2}{dt} = k_{AB}\,[A_2][B] - k_{XA}\,[A][X] = 0 \qquad K_{eq,2} = \left(\frac{[A][X]}{[A_2][B]}\right)_{eq} \equiv \frac{k_{AB}}{k_{XA}} \qquad 4.5.8$$

$$\frac{1}{V}\frac{d\xi_3}{dt} = k_{X}\,[X] - k_{DC}\,[C][D] = 0 \qquad K_{eq,3} = \left(\frac{[C][D]}{[X]}\right)_{eq} \equiv \frac{k_{X}}{k_{DC}} \qquad 4.5.9$$

where ξ_1, ξ_2, and ξ_3 are the extents of each step. The mechanistic steps for the forward processes add to give the overall reaction stoichiometry, therefore the product of the equilibrium constants for the individual mechanistic steps always gives the overall equilibrium constant:

$$K_{eq,1}\,K_{eq,2}\,K_{eq,3} = \left(\frac{[A_2][M]}{[A]^2[M]}\right)_{eq}\left(\frac{[A][X]}{[A_2][B]}\right)_{eq}\left(\frac{[C][D]}{[X]}\right)_{eq} = \left(\frac{[C][D]}{[A][B]}\right)_{eq} = K_{eq} \qquad 4.5.10$$

Since $K_{eq,1}$, $K_{eq,2}$, and $K_{eq,3}$ are each constants, the overall K_{eq} must also be a constant. In other words, the overall equilibrium ratio of products to reactants is given by the overall reaction stoichiometry and is independent of the molecularity of the intervening mechanistic steps. Substituting Eq. 4.5.7-4.5.9 into Eq. 4.5.10, the overall equilibrium constant is then given by:

$$K_{eq} = \frac{k_{AM}}{k_{MA}} \frac{k_{AB}}{k_{XA}} \frac{k_X}{k_{DC}} \qquad 4.5.11$$

which has all the forward rate constants in the numerator and all the reverse rate constants in the denominator. We derived Eq. 4.5.11 for a specific mechanism. However, notice that the final results will be true for any number of steps with any molecularity. We just chose a specific example to make the derivation less abstract. A more general expression is:

$$A + B \underset{k_{-1}}{\overset{k_1}{\rightleftarrows}} C + D \underset{k_{-2}}{\overset{k_2}{\rightleftarrows}} E + F \underset{k_{-3}}{\overset{k_3}{\rightleftarrows}} G + H \; ....$$

with $$K_{eq} = \frac{k_1\, k_2\, k_3 \ldots.}{k_{-1}\, k_{-2}\, k_{-3} \ldots.} \qquad 4.5.12$$

Eqs. 4.5.11 and 4.5.12 tell us something else quite important. In setting up a mechanism, not all the rate constants and the overall equilibrium constant are independent. In most experimental circumstances, the equilibrium constant is known from equilibrium measurements, but it is often difficult to measure the rate constants for all the mechanistic steps. For example, in Eq. 4.5.11, once the overall equilibrium constant is measured and five of the six rate constants have been determined, the sixth can be calculated using the overall equilibrium ratio.

Cyclic Reaction Mechanisms Have an Important Constraint: We began this section by considering a cyclic mechanism. You might suspect that cyclic mechanisms are uncommon. However, cyclic mechanisms of the general type in Eq. 4.5.2 are quite common, especially in biochemistry.[10,11] Detailed balance is an important principle that helps guide the proper construction of complicated, complete mechanisms of all types. Two examples of cyclic reaction mechanisms are shown in Figure 4.5.1. Figure 4.5.1a is an enzyme mechanism when the product forms a stable complex with the enzyme. Figure 4.5.1b shows the processes that are active in aqueous solutions of weak acids. For weak acid proton transfer, all these steps, which include the weak acid dissociation, conjugate base hydrolysis, and auto-protolysis of water, are needed to understand kinetics experiments in weak acids. For cyclic mechanisms in general, each cycle in a mechanism provides an additional constraint on the set of rate constants.

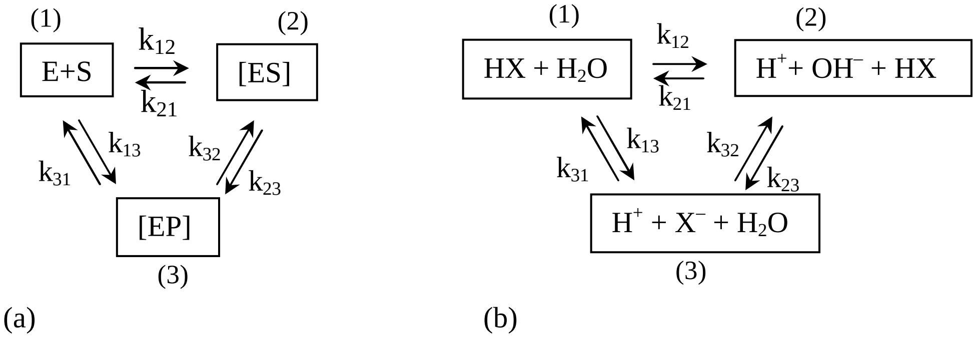


Figure 4.5.1: Two examples of three-state cyclic reaction mechanisms. (a) The Michaelis-Menten mechanism is extended to include the formation of an enzyme-product complex. This mechanism is one example of product inhibition. (b) The dissociation of a weak acid in aqueous solution, for example acetic acid.[12] For this mechanism $K_w = k_{12}/k_{21}$, $K_a = k_{13}/k_{31}$, and for the conjugate base, $K_b = k_{32}/k_{23}$.

Consider the cyclic reaction in Eq. 4.5.2. The equilibrium constants for each of the mechanistic steps, taken in the order A → B → C → A, are:

$$K_1 = \left(\frac{[B]}{[A]}\right)_{eq} \qquad K_2 = \left(\frac{[C]}{[B]}\right)_{eq} \qquad K_3 = \left(\frac{[A]}{[C]}\right)_{eq} \tag{4.5.13}$$

Notice that the product of the equilibrium constants around the cycle gives:

$$K_1\, K_2\, K_3 = \left(\frac{[B]}{[A]}\right)_{eq}\left(\frac{[C]}{[B]}\right)_{eq}\left(\frac{[A]}{[C]}\right)_{eq} = 1 \tag{4.5.14}$$

In other words, the net process is $A \rightleftarrows A$, which has an equilibrium constant of one. Following Eq. 4.5.12, we can also do the product of the equilibrium constants in terms of the forward and reverse rate constants around the cycle:

$$K_1 = \frac{k_{AB}}{k_{BA}} \qquad K_2 = \frac{k_{BC}}{k_{CB}} \qquad K_3 = \frac{k_{CA}}{k_{AC}} \tag{4.5.15}$$

giving:

$$K_1\, K_2\, K_3 = \frac{k_{AB}\, k_{BC}\, k_{CA}}{k_{BA}\, k_{CB}\, k_{AC}} = 1 \tag{4.5.16}$$

In other words, the product of all the forward rate constants divided by the product of all the reverse rate constants is equal to one. The ramification of this result is that for cyclic mechanisms not all the rate constants are independent. For this example, once five rate constants are specified, the sixth is to be calculated from Eq. 4.5.16. For reactions with more than one cycle, there is one dependent rate constant per cycle.[9,11,13]

Example 4.5.1: *Cyclic Mechanisms*
How many independent rate constants are needed to determine the kinetics of proton exchange in aqueous solutions of acetic acid if two equilibrium constants are known? The mechanism is given in Figure 4.5.1b. The K_a for acetic acid is $1.75\text{x}10^{-5}$ and K_w is $1.008\text{x}10^{-14}$ (or alternatively if the concentration of water is treated explicitly, $K_c = [H_2O]_{eq}/K_w = 5.489\text{x}10^{15}$ at 298.2 K; see Sec. 3.6). Give the relationships that relate the rate constants to each other.

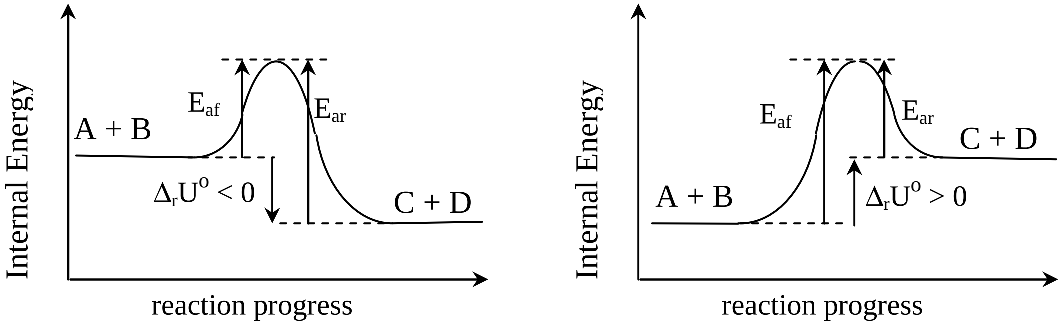
Answer: Because of the cyclic mechanism, the number of independent rate constants is five. The relationship around the cycle is based on Eq. 4.5.16:

$$\frac{k_{12}\, k_{23}\, k_{31}}{k_{21}\, k_{32}\, k_{13}} = 1$$

The specification of the acid dissociation constant, $K_a = k_{13}/k_{31}$, relates k_{13} and k_{31}. The auto-protolysis equilibrium constant for water relates k_{12} and k_{21}. The result is that <u>three</u> independent rate constants need to be determined. If the water concentration is included in the equilibrium constant, $K_w = k_{12}/k_{21}$. [Note that, if the concentration of water is treated explicitly, $K_c = k_{21}/k_{12}$ (see Sec. 3.6).]

Catalysis Doesn't Change the Equilibrium Constant: Another example of a reaction mechanism that follows the form of Eq. 4.5.1 is a reaction that is catalyzed through the formation of a reactive intermediate. Consider the overall reaction $A \rightleftarrows C$, with the mechanism $A \rightarrow B \rightarrow C$ as a catalyzed path in the forward direction and $C \rightarrow A$ an uncatalyzed path in the reverse direction. Is such a mechanism possible? One important ramification of detailed balance is that the forward and reverse mechanism for a catalyzed reaction must be the same. In other words, if the forward step is catalyzed, so must the reverse step. The proper mechanism for a catalyzed path and a parallel uncatalyzed path is then Eq. 4.5.2. The catalyzed and uncatalyzed processes occur in parallel. The net result is that the equilibrium constant for a catalyzed reaction is unchanged from the uncatalyzed reaction; both paths must have the same equilibrium constant since they connect the same two states, A and C. We should look at the effects of adding a catalyst in more detail.

Consider the reaction progress for typical single-step reactions, Figure 4.5.2. The horizontal axis is the reaction progress. The reaction progress corresponds to a concerted reactive asymmetric stretch that corresponds to stretching the bonds that are broken and contracting the bonds that are formed. This motion brings about the bond breaking and making steps for the reaction. The peak in the reaction profile corresponds to the formation of the transition state. The transition state is an unstable point on the reaction path that exists for a fleetingly short time, often only a few femtoseconds ($1 \text{ fs} = 1\text{x}10^{-15}$ s). Assume that the reaction is taking place at constant temperature and in a constant volume vessel. Remember from General Chemistry that the internal energy change for the reaction, $\Delta_r U^o$, is the appropriate measure of the energetics of the reaction at constant temperature and volume. Note that General Chemistry texts use ΔE for internal energy changes, but the international standard is to use ΔU, instead. For an exothermic reaction the change in reaction internal energy is negative. For an endothermic reaction the change in reaction internal energy is positive. The energy necessary to reach the transition state from the reactants is E_{af} and the energy necessary to reach the transition state from the products is E_{ar}.



(a) Exothermic reaction (b) Endothermic reaction

Figure 4.5.2: Reaction profile for an (a) exothermic and (b) endothermic reaction at constant temperature and volume.

Using Figure 4.5.2, the activation energies for the forward and reverse processes are related through the reaction internal energy:

$$E_{af} - E_{ar} = \Delta_r U^o \qquad \text{(cst. V)} \qquad 4.5.17$$

Also from General Chemistry, remember that the enthalpy change for the reaction, $\Delta_r H^o$, is the appropriate measure of the energetics of the reaction at constant temperature and pressure. If the reaction is bimolecular in both directions or takes place in solution, the difference in activation energies is also equal to the change in reaction enthalpy:

$$E_{af} - E_{ar} = \Delta_r H^o \qquad \text{(cst. P or bidirectional-bimolecular or solution)} \qquad 4.5.18$$

The equilibrium constant for the reaction is given by the ratio of the forward and reverse rate constants, which in turn are given by the corresponding Arrhenius expressions:

$$K_{eq} = \frac{k_f}{k_r} = \frac{A_f\, e^{-E_{af}/RT}}{A_r\, e^{-E_{ar}/RT}} \qquad 4.5.19$$

where A_f and A_r are the pre-exponential factors for the forward and reverse reactions, respectively.

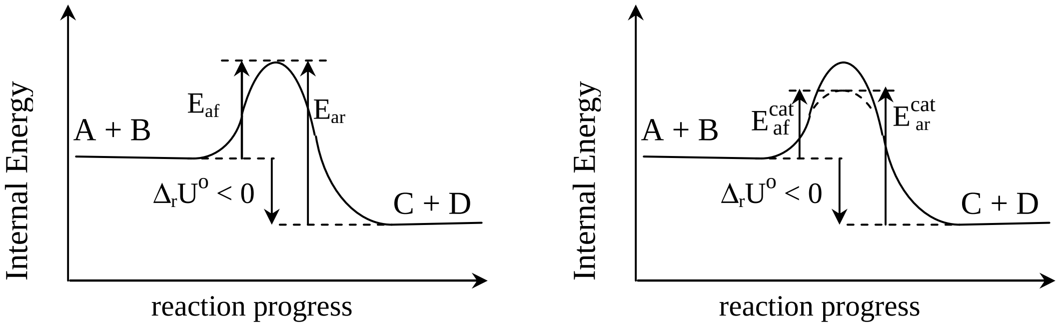
Catalysts act on elementary steps by decreasing the activation energy or by increasing the pre-exponential factor. We will return to reaction profiles and catalysis after we have covered statistical mechanics, at which point we will be able to be much more specific and complete in our description of reaction profiles. However, for now, we wish to focus on the activation energy and the reaction profile to help understand the interaction between catalysis and the equilibrium constant for a single-step reaction.

Assume that a catalyst lowers the activation energy for the forward process by an amount ε:

$$E_{af}^{cat} = E_{af} - \varepsilon \qquad 4.5.20$$

Then from Figure 4.5.3, the activation energy for the reverse process must also be lowered by the same amount:

$$E_{ar}^{cat} = E_{ar} - \varepsilon \qquad 4.5.21$$



(a) Original reaction (b) Catalyzed reaction

Figure 4.5.3: Reaction profile for an (a) uncatalyzed and (b) catalyzed reaction. If a catalyst lowers the activation energy for the forward process, the activation energy for the reverse process is also lowered by the same amount.

Assuming Arrhenius temperature dependence, using Eq. 3.5.1, the forward rate is accelerated to:

$$k_f^{cat} = A_f\, e^{-(E_{af} - \varepsilon)/RT} \qquad 4.5.22$$

and the reverse rate is accelerated to:

$$k_r^{cat} = A_r\, e^{-(E_{ar} - \varepsilon)/RT} \qquad 4.5.23$$

where k_f^{cat} and k_r^{cat} are the rate constants for the catalyzed reaction. However, notice that the equilibrium constant, which is the ratio of the forward to reverse rate constants, remains unchanged:

$$K_{eq}^{cat} = \frac{k_f^{cat}}{k_r^{cat}} = \frac{A_f\, e^{-(E_{af} - \varepsilon)/RT}}{A_r\, e^{-(E_{ar} - \varepsilon)/RT}} = \frac{A_f\, e^{-E_{af}/RT}\, e^{\varepsilon/RT}}{A_r\, e^{-E_{ar}/RT}\, e^{\varepsilon/RT}} = \frac{A_f\, e^{-E_{af}/RT}}{A_r\, e^{-E_{ar}/RT}} = K_{eq} \qquad 4.5.24$$

This result is a consequence of detailed balance. Detailed balance can also help with construction of rate laws.

Activation Energies for Elementary Steps Combine to Give the Overall Activation Energy: We mentioned in Chapter 3 that the activation energy for a reaction can be a negative number. How can that happen? Another important and related question is how does the overall activation energy for a reaction depend on the activation energies for the elementary steps in the mechanism? To answer these questions, we first assume Arrhenius behavior for the overall reaction and each elementary step. From Eq. 3.5.3, taking the temperature derivative of both sides of the equation at constant volume gives:

$$\frac{d \ln k}{dT} = \frac{E_a}{RT^2} \qquad \text{(cst. V)} \qquad 4.5.25$$

for the overall reaction. This equation can also be applied to each individual elementary step. Consider the S_N1 mechanism as a typical multi-step mechanism. For the case given in Eq. 4.2.17, the rate law has the form:

$$\frac{d[P]}{dt} = k\, \frac{[RX]\,[Nuc:^-]}{[X^-]} \qquad (k_{-1}\,[X^-] >> k_2[Nuc:^-]) \qquad 4.5.26$$

with the effective rate constant, $k = k_2\, k_1/k_{-1}$. We can then find the overall activation energy using Eq. 4.5.25 for each elementary rate constant:

$$\frac{d \ln k}{dT} = \frac{d \ln k_2}{dT} + \frac{d \ln k_1}{dT} - \frac{d \ln k_{-1}}{dT} = \frac{E_{a2}}{RT^2} + \frac{E_{a1}}{RT^2} - \frac{E_{a-1}}{RT^2} \qquad 4.5.27$$

where E_{a1} is the activation energy for the forward direction for the first step, E_{a-1} is the activation energy for the reverse of the first step, and E_{a2} is the activation energy for the formation of product. Comparing Eq. 4.5.27 to Eq. 4.5.25 shows that the overall activation energy for the reaction is $E_a = E_{a2} + E_{a1} - E_{a-1}$. It is easy to see that the overall activation energy may be negative even though the activation energy for each elementary step is positive. We can use detailed balance to relate the overall activation energy to Le Chatelier's principle. Eq. 4.2.17 or Eq. 4.5.27 corresponds to the pre-equilibrium mechanism, with $K_c = k_1/k_{-1}$, giving the effective rate constant as $k = k_2\, k_1/k_{-1} = k_2\, K_c$. Using Eq. 4.5.17 then relates the overall activation energy to the reaction internal energy:

$$E_a = E_{a2} + E_{a1} - E_{a-1} = E_{a2} + \Delta_r U^o \qquad \text{(cst. V)} \qquad 4.5.28$$

Thus, we find the overall activation energy is the sum of the activation energy for the slow step and the internal energy change for the pre-equilibrium step. If the equilibrium step is more exothermic than the activation energy, $-\Delta_r U° > E_{a2}$, then the overall activation energy will be negative. In other words, using La Chatelier's principle, the pre-equilibrium step will shift in the endothermic direction with an increase in temperature. If the pre-equilibrium is sufficiently exothermic, the overall reaction will slow as the temperature increases because the equilibrium constant for the pre-equilibrium step decreases.

This same general method, based on Eq. 4.5.25, can be used whenever the overall rate law can be expressed with a single rate constant. The method is general, assuming Arrhenius behavior, but the specific results depend on the details of the mechanism. The overall activation energy for such mechanisms is then seen to be a simple algebraic combination of the activation energies of the elementary steps.[14]

Example 4.5.2: *Overall Activation Energy of a Multi-Step Mechanism*
The general chain mechanism for the $H_2 + Br_2$ reaction will not show Arrhenius behavior because of the rate constants in the denominator of the rate law in Eq. 4.3.16. However, if the inhibition step is slow, the rate law simplifies to:

$$\frac{d[HBr]}{dt} = 2\, k_2\, k_1^{½}\, k_5^{-½}\, [H_2][Br_2]^{½}$$

(a) Find the relationship of the overall activation energy to the activation energies for the elementary steps in the mechanism. (b) Under what circumstances might this slow inhibition approximation be valid?

Answer: (a). The effective rate constant is $k = 2\, k_2\, k_1^{½}\, k_5^{-½}$. Then taking the logarithm gives:

$$\ln k = \ln 2 + \ln k_2 + ½ \ln k_1 - ½ \ln k_5$$

And taking the temperature derivative

$$\frac{d \ln k}{dT} = \frac{d \ln k_2}{dT} + ½ \frac{d \ln k_1}{dT} - ½ \frac{d \ln k_5}{dT} = \frac{E_{a2}}{RT^2} + ½ \frac{E_{a1}}{RT^2} - ½ \frac{E_{a5}}{RT^2}$$

Which gives $E_a = E_{a2} + ½ E_{a1} - ½ E_{a5}$

(b). Slow inhibition would result if k_4 is small or at the beginning of the reaction when $[Br_2]$ is large and [HBr] is small. In this second case, the activation energy in part (a) would be the expected result from an initial rate study.

Detailed balance can also be helpful in postulating the rate laws for the reverse process of reversible steps.

Detailed Balance Provides Some Possible Mechanisms for Reverse Processes: Initial rate studies often provide the rate law for the forward process for a reaction. As the reaction progresses, reverse processes become more important, until at equilibrium the forward rate is equal to the reverse rate. Can detailed balance be used to find the rate law for the reverse process if the rate law for the forward process is known? Detailed balance can be used to suggest some possible rate laws, but no single rate law may be obtained. The reason is that there is some ambiguity in how we write the overall reaction. Take for example the oxidation of nitric oxide. We can write the reaction with a variety of stoichiometric coefficients, which give a variety of corresponding equilibrium constants. Two examples are:

$$2\ \mathrm{NO}\ (g) + \mathrm{O_2}(g) \rightleftarrows 2\ \mathrm{NO_2}\ (g) \qquad K_c = \left(\frac{[\mathrm{NO_2}]^2}{[\mathrm{NO}]^2[\mathrm{O_2}]}\right)_{eq} \qquad \text{(equilibrium)} \quad 4.5.29$$

$$\text{or} \qquad \mathrm{NO}\ (g) + ½\ \mathrm{O_2}(g) \rightleftarrows \mathrm{NO_2}\ (g) \qquad K_c{}' = \left(\frac{[\mathrm{NO_2}]}{[\mathrm{NO}][\mathrm{O_2}]^{½}}\right)_{eq} \qquad \text{(equilibrium)} \quad 4.5.30$$

where $K_c{}' = K_c{}^{½}$. Of course, the equilibrium position of the reaction doesn't change as we change the overall stoichiometric coefficients; we just express the equilibrium condition in a different functional form. We need to take this ambiguity into account as we relate forward and reverse processes. In general, from Eq. 4.5.29 we can write:

$$K_c{}^{S} = \left(\frac{[\mathrm{NO_2}]^2}{[\mathrm{NO}]^2[\mathrm{O_2}]}\right)_{eq}^{s} \qquad \text{(equilibrium)} \quad 4.5.31$$

where the coefficient, s, accounts for the different ways that we might choose to write the overall reaction. The coefficient s can be a positive integer or simple fraction.[15] For Eq. 4.5.30, s = ½. While there is ambiguity in how we express the equilibrium constant, the rate laws are fixed and determined directly from experiment. First notice that by cross multiplication, Eq. 4.5.31 can be rearranged to give:

$$\left[\left(\frac{[\mathrm{NO}]^2[\mathrm{O_2}]}{[\mathrm{NO_2}]^2}\right)_{eq} K_c\right]^{s} = 1 \qquad \text{(equilibrium)} \quad 4.5.32$$

Correspondingly, for the general reaction, a A + b B $\rightleftarrows$ c C + d D, we can write:[15]

$$K_c = \left(\frac{[C]^c[D]^d}{[A]^a[B]^b}\right)_{eq} \qquad \text{and} \qquad \left[\left(\frac{[A]^a[B]^b}{[C]^c[D]^d}\right) K_c\right]^{s}_{eq} = 1 \qquad \text{(equilibrium)} \quad 4.5.33$$

If the forward rate process has an experimentally determined rate law in the form:

$$\upsilon_+ = k_f\,[A]^{\alpha_f}\,[B]^{\beta_f}\,[C]^{\chi_f}\,[D]^{\delta_f} \qquad 4.5.34$$

The rate law for the reverse process will be in the form:

$$\upsilon_- = k_r\,[A]^{\alpha_r}\,[B]^{\beta_r}\,[C]^{\chi_r}\,[D]^{\delta_r} \qquad 4.5.35$$

We use Greek letters for the kinetic coefficients and regular font for the equilibrium law coefficients. At equilibrium, the forward and reverse rates are equal giving the ratio $\upsilon_+/\upsilon_- = 1$, and then the ratio of Eq. 4.5.34 to 4.5.35 is related to Eq. 4.5.33 by:

$$\frac{k_f}{k_r}[A]^{\alpha_f-\alpha_r}\,[B]^{\beta_f-\beta_r}\,[C]^{\chi_f-\chi_r}\,[D]^{\delta_f-\delta_r} = \left[\left(\frac{[A]^a[B]^b}{[C]^c[D]^d}\right)_{eq} K_c\right]^{s} = 1 \qquad \text{(equilibrium)} \quad 4.5.36$$

Eq. 4.5.36 is a sufficient, but not necessary condition; other more complex mechanisms may be possible.[16] Comparing term by term for each reactant and product, Eq. 4.5.36 is satisfied if:

$$\begin{aligned} \alpha_f - \alpha_r &= as \\ \beta_f - \beta_r &= bs \\ \chi_f - \chi_r &= -cs \\ \delta_f - \delta_r &= -ds \\ \text{and } \frac{k_f}{k_r} &= K_c^{\,s} \end{aligned} \qquad 4.5.37$$

Various possible values for s give alternative possibilities for the rate law for the reverse process. Note, however, that Eqs. 4.5.37 only hold if the rate law for the forward reaction is found to follow the simple form in Eq. 4.5.34.

Example 4.5.3: *Rate Law for a Reversible Mechanism from Detailed Balance*
For the important atmospheric reaction: $2\ \mathrm{NO}\ (g) + \mathrm{O_2}(g) \rightleftarrows 2\ \mathrm{NO_2}\ (g)$

the rate law as determined in the laboratory for the early stages of the reaction is:

$$\upsilon_+ = -\frac{1}{2}\frac{d[\mathrm{NO}]}{dt} = k_f\,[\mathrm{NO}]^2\,[\mathrm{O_2}] \qquad \text{(initial rate law)}$$

Find two possible rate laws for the reverse process, and write the two corresponding final total rate laws that would apply as the system approaches equilibrium.

Answer: The corresponding equilibrium expression is:

$$K_c = \left(\frac{[\mathrm{NO_2}]^2}{[\mathrm{NO}]^2[\mathrm{O_2}]}\right)_{eq}$$

First assume s = 1. From the given forward rate law $\alpha_f = 2$, $\beta_f = 1$, $\chi_f = 0$. Then the conditions from Eq. 4.5.37 are:

$$\begin{aligned} 2 - \alpha_r &= 2(1) &\quad \text{or} \quad& \alpha_r = 0 \\ 1 - \beta_r &= 1(1) &\quad \text{or} \quad& \beta_r = 0 \\ 0 - \chi_r &= -2(1) &\quad \text{or} \quad& \chi_r = 2 \\ \text{and } \frac{k_f}{k_r} &= K_c \end{aligned}$$

or the rate law for the reverse reaction is $\upsilon_- = k_r\,[\mathrm{NO_2}]^2$
The final overall rate law is:

$$\upsilon = -\frac{1}{2}\frac{d[\mathrm{NO}]}{dt} = k_f\,[\mathrm{NO}]^2\,[\mathrm{O_2}] - k_r\,[\mathrm{NO_2}]^2 \qquad (s = 1)$$

Now assume s = ½. Then the conditions from Eq. 4.5.37 are:

$$\begin{aligned} 2 - \alpha_r &= 2(½) &\quad \text{or} \quad& \alpha_r = 1 \\ 1 - \beta_r &= 1(½) &\quad \text{or} \quad& \beta_r = ½ \qquad \text{and } \frac{k_f}{k_r} = K_c^{\,½} \\ 0 - \chi_r &= -2(½) &\quad \text{or} \quad& \chi_r = 1 \end{aligned}$$

or the rate law for the reverse reaction is $\upsilon_- = k_r'\,[\mathrm{NO}][\mathrm{O_2}]^{½}[\mathrm{NO_2}]$

The final overall rate law is:

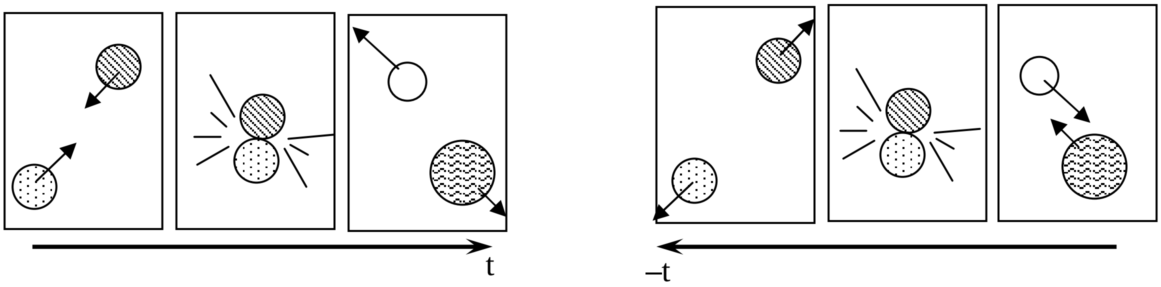
$$\upsilon = -\frac{1}{2}\frac{d[NO]}{dt} = k_f\,[NO]^2\,[O_2] - k_r'\,[NO][O_2]^{½}[NO_2] \qquad (s = ½)$$

As a final check, we should calculate the corresponding equilibrium constant from the last rate law. Setting the last equation equal to zero for the equilibrium state gives:

$$K_c' = \frac{k_f}{k_r'} = \left(\frac{[NO][O_2]^{½}[NO_2]}{[NO]^2\,[O_2]}\right)_{eq} = \left(\frac{[NO_2]}{[NO]\,[O_2]^{½}}\right)_{eq} = K_c^{½}$$

as required by Eq. 4.5.29-4.5.31.

Detailed Balance is a Consequence of Microscopic Reversibility: Why is detailed balance required for all elementary steps in a mechanism? Remember that elementary steps describe the collisions that occur in the gas phase, or the encounters that occur in solution. Consider a collision as described by classical mechanics. The laws of classical mechanics are symmetric with respect to the direction of time. If every t in Newton's Laws is replaced by –t, the equations of motion are still valid; the energy of a given configuration is unchanged. For example, consider the single collision in Figure 4.5.4a moving in the forward direction of time from left to right. If, after the collision, the momenta of all the particles are reversed and the initial and final states are reversed, the collision will reoccur exactly as it did before giving back reactants, as if time were running backwards. In other words, if we replace each momentum, $\vec{p} = -\vec{p}$, the collision will reverse and give back reactants with reversed momenta. However, $\vec{p}$ and $-\vec{p}$ are equally likely because they give the same kinetic energy, which is $E_k = p^2/2m$. Because Newton's Laws don't differ with the direction of time, the collision is equally probable in either direction. This result is called **microscopic reversibility**.



(a). (b).

Figure 4.5.4: A collision is equally likely from either direction, reactants → products or products → reactants. (a). The original collision moving from left to right. (b). The collision after the momenta have been replaced, $\vec{p} = -\vec{p}$, with the progression running from right to left, t = –t.

Detailed balance is a consequence of microscopic reversibility. Microscopic reversibility holds for every collision. Each mechanistic step is equally probable with its exact opposite, giving that at equilibrium the forward rate is equal to the reverse rate for each mechanistic step. However, microscopic reversibility is more general than detailed balance, because microscopic reversibility holds for each collision throughout the time course of the reaction, while detailed balance only holds at equilibrium. Advanced theories of molecular collisions are based on quantum mechanics. Quantum mechanics is also symmetrical with respect to the direction, or sign, of time. Microscopic reversibility holds equally well for quantum mechanical and classical systems.

The theory of reaction rates that we have been studying is based directly on experimental observation; we can call this theory **empirical** kinetics. Neither microscopic reversibility nor detailed balance is required by the theories of empirical chemical kinetics and thermodynamics. Detailed balance is a requirement that is placed upon empirical kinetics and thermodynamics by our interpretation of chemical events on a molecular basis. Because of this distinction, detailed balance is said to be an **extra-thermodynamic** requirement. Even though we have used a molecular view in the last two chapters, neither empirical kinetics nor thermodynamics requires a molecular interpretation.

4.6 Summary – Looking Ahead

Complicated mechanisms are composed of parallel, consecutive, and reversible steps. A proposed mechanism must agree with the empirical rate law. This agreement can be checked by predicting the overall rate law from the mechanism or by comparing the integrated rate laws for the elementary steps with the experimental time course. For multi-step mechanisms, the steady-state approximation is useful for predicting the overall rate law. When comparing the integrated rate laws for the elementary steps to the experimental time course, the finite difference approximation can be used instead of analytical integrals. Chain mechanisms are a common type of multi-step mechanism in environmental and biochemical processes. Autocatalytic processes far from equilibrium can generate temporal and spatial oscillations.

For chain mechanisms, the chain initiation step is often the critical step. The rate of the chain initiation step for the $H_2 + Br_2$ reaction, Eq. 4.3.3, is negligible at room temperature. The chain initiation step for the $H_2 + O_2$ reaction, Eq. 4.3.20, is also negligible at room temperature in the absence of a catalyst. This slow initiation step is the reason for the kinetic stability of H_2 and O_2 mixtures. After the initiation step, autocatalyic processes rapidly accelerate the reaction. Often chain initiation steps are driven by the absorption of light to speed the initiation process. Light is a convenient source of energy to help overcome the activation energy barrier for difficult processes. Many environmental processes are driven by sunlight. For example, important redox intermediates in aquatic environmental chemistry that result from photolytic processes include peroxide, superoxide, and hydroxyl radicals. Another way to increase the rate of the chain initiation steps is to provide a catalyst. Heterogeneous catalysis is mediated through interactions on surfaces. In the next chapter we discuss the kinetics of photochemical and surface reactions.

Chapter Summary

1. A mechanism is a sequence of elementary steps.
2. A proposed mechanism should result in a predicted overall rate law that has the same concentration dependences as the experimentally determined rate law.
3. The rate law for each elementary step is directly determined from the molecularity, because the elementary steps describe the collisions that take place.
4. Alternate mechanisms that agree with the experimental rate law are kinetically equivalent.

5. An intermediate is a species that is neither a reactant nor a product.
6. Proving that a mechanism is the correct mechanism is often compared to building a case in a court of law; you can never be sure that a proposal is absolutely and exclusively correct.
7. For two first-order parallel unidirectional reactions: $A \xrightarrow{k_1} B$, $A \xrightarrow{k_2} C$:

$$[A] = [A]_o\, e^{-(k_1+k_2)\,t}\,, \quad [B] = \frac{k_1\,[A]_o}{(k_1 + k_2)}\,(1 - e^{-(k_1+k_2)\,t}), \quad [C] = \frac{k_2\,[A]_o}{(k_1 + k_2)}\,(1 - e^{-(k_1+k_2)\,t})$$

and the ratio of the two products does not depend on time: $[B]/[C] = k_1/k_2$

8. For multiple first-order parallel unidirectional steps with $\tau_1 = 1/k_1$, $\tau_2 = 1/k_2$, etc., the lifetime of the reaction is:

$$\frac{1}{\tau_{obs}} = \frac{1}{\tau_1} + \frac{1}{\tau_2} + \frac{1}{\tau_3} + \ldots.$$

9. The finite difference approximation for a first-order rate law is: $[A](t+\Delta t) = [A](t) - k[A](t)\,\Delta t$
10. A first-order parallel mechanism with reversible steps, $A \rightleftarrows X$, $A \rightleftarrows Y$, has equilibrium constants $K_{c1} = k_1/k_{-1}$ and $K_{c2} = k_2/k_{-2}$. The kinetic product is favored at short times and the thermodynamic product is favored at long times.
11. For a first-order consecutive unidirectional mechanism: $A \xrightarrow{k_1} B \xrightarrow{k_1'} C$:

$$[A] = [A]_o\, e^{-k_1 t}, \quad [B] = [A]_o\left(\frac{k_1}{k_1' - k_1}\right)(e^{-k_1 t} - e^{-k_1' t})\,,$$

$$[C] = [A]_o\left(1 + \left(\frac{1}{k_1 - k_1'}\right)\left(k_1'\, e^{-k_1 t} - k_1\, e^{-k_1' t}\right)\right)$$

which for $k_1 << k_1'$ reduces to $[C] = [A]_o(1 - e^{-k_1 t})$. The intrinsically slow step is the rate determining step.

12. The steady-state approximation cannot be applied if the concentrations of the intermediates are not much less than the initial concentration of reactants. The steady-state approximation should not be applied to reactants or products. The steady-state approximation does not hold at the beginning of the reaction if the reaction is not initially at equilibrium.
13. The mechanism for S_N1 nucleophilic substitution reactions is:

$$RX \underset{k_{-1}}{\overset{k_1}{\rightleftarrows}} R^+ + X^- \qquad\qquad R^+ + Nuc{:}^- \xrightarrow{k_2} R\text{-}Nuc$$

Assuming the steady-state approximation gives: $\dfrac{d[P]}{dt} = \dfrac{k_2\, k_1\,[RX]\,[Nuc{:}^-]}{k_{-1}\,[X^-] + k_2[Nuc{:}^-]}$ $(k_1 << k_2)$

14. For the S_N1 mechanism at the beginning of reaction when $[X^-]$ is small and $[Nuc{:}^-]$ is large, the rate law reduces to a first-order process:

$$\frac{d[P]}{dt} = k_1\,[RX] \qquad\qquad (k_{-1}\,[X^-] << k_2[Nuc{:}^-])$$

At the end of the reaction when $[X^-]$ is large and $[Nuc{:}^-]$ is small, the rate law reduces to the pre-equilibrium result:

$$\frac{d[P]}{dt} = \frac{k_2\, k_1\, [RX]\, [Nuc{:}^-]}{k_{-1}\, [X^-]} \qquad (k_{-1}\, [X^-] >> k_2[Nuc{:}^-])$$

15. If the steady-state approximation applies, all mechanistic steps have the same net rate.
16. The Michaelis-Menten mechanism is:

$$E + S \; \underset{k_{-1}}{\overset{k_2}{\rightleftarrows}} \; ES \; \xrightarrow{k_1} \; P + E$$

Applying the steady-state approximation gives: $\frac{d[P]}{dt} = k_1 \frac{k_2\, [E]_o[S]}{(k_{-1} + k_1 + k_2[S])}$ $\quad (k_2 << k_1)$

17. The maximum rate for the Michaelis-Menten mechanism is $\upsilon_{max} = k_1\, [E]_o$ and the Michaelis constant is defined as $k_M \equiv \frac{(k_1 + k_{-1})}{k_2}$ giving the initial rate: $\frac{1}{\upsilon_o} = \frac{1}{\upsilon_{max}} + \frac{k_M}{\upsilon_{max}\, [S]_o}$
18. The Lindemann-Henshelwood mechanism for unimolecular reactions is:

$$A + A \underset{k_{-2}}{\overset{k_2}{\rightleftarrows}} A + A^* \qquad\qquad A^* \; \xrightarrow{k_1} \; B + C$$

Applying the steady-state approximation gives: $\frac{d[B]}{dt} = k_1[A^*] = \frac{k_1 k_2[A]^2}{k_{-2}[A] + k_1}$ $\quad (k_2 << k_1)$

19. For the Lindemann-Henshelwood mechanism, in the rapid deactivation limit $k_1 << k_{-2}[A]$, the rate law becomes first-order:

$$\frac{d[B]}{dt} = \frac{k_1 k_2}{k_{-2}}[A] \qquad (k_2 << k_1,\ k_1 << k_{-2}[A])$$

For slow deactivation when $k_{-2}[A] << k_1$ the rate law reduces to:

$$\frac{d[B]}{dt} = k_2[A]^2 \qquad (k_2 << k_1,\ k_{-2}[A] << k_1)$$

20. A chain mechanism begins with the formation of a chain carrier in a chain initiation step. Chain propagation steps both consume and produce chain carriers. A chain branching step is a chain propagation step that increases the net number of chain carriers. Chain propagation steps are in competition with chain breaking steps that consume reactive intermediates and produce stable reactants or products.
21. The rate of an autocatalytic process increases with the production of products. The product is also a reactant for one or more mechanistic steps.
22. Autocatalytic processes show an induction period, followed by a rapid rise in rate as products begin to build, but then finally the rate slows as a reactant is exhausted.
23. Autocatalysis is an example of a general phenomenon called positive feedback.
24. Oscillating reactions have autocatalytic mechanisms and are far from equilibrium.
25. Detailed balance requires that for a reaction at equilibrium, the forward and reverse reaction rates for each elementary step in the mechanism must be equal.
26. A complete mechanism includes a series of steps and the exact reverse for each step.
27. The sum of the forward processes in a complete mechanism must give the overall reaction stoichiometry.

28. For the general mechanism at equilibrium:

$$\mathrm{A + B} \underset{k_{-1}}{\overset{k_1}{\rightleftarrows}} \mathrm{C + D} \underset{k_{-2}}{\overset{k_2}{\rightleftarrows}} \mathrm{E + F} \underset{k_{-3}}{\overset{k_3}{\rightleftarrows}} \mathrm{G + H} \;....$$

detailed balance gives: $K_{eq} = \dfrac{k_1\, k_2\, k_3 \ldots.}{k_{-1}\, k_{-2}\, k_{-3} \ldots.}$

29. Each cycle in a mechanism provides an additional constraint on the set of rate constants. For

$A \rightleftarrows B \rightleftarrows C \rightleftarrows A$: $K_1\, K_2\, K_3 = \left(\dfrac{[B]}{[A]}\right)_{eq} \left(\dfrac{[C]}{[B]}\right)_{eq} \left(\dfrac{[A]}{[C]}\right)_{eq} = 1$

30. Catalysis doesn't change the equilibrium constant for a reaction.

31. Assuming Arrhenius behavior, at constant volume: $\dfrac{d \ln k}{dT} = \dfrac{E_a}{RT^2}$

holds for the overall reaction and each individual elementary step. Assuming Arrhenius behavior, the overall activation energy is a simple algebraic combination of the activation energies of the elementary steps.

32. Detailed balance provides possible mechanisms for reverse processes. For the reaction $aA + bB \rightleftarrows cC + dD$, if the forward process has a rate law, $\upsilon_+ = k_f\, [A]^{\alpha_f}\, [B]^{\beta_f}\, [C]^{\chi_f}\, [D]^{\delta_f}$, the reverse process will give $\upsilon_- = k_r\, [A]^{\alpha_r}\, [B]^{\beta_r}\, [C]^{\chi_r}\, [D]^{\delta_r}$ with:

$\alpha_f - \alpha_r = as$
$\beta_f - \beta_r = bs$
$\chi_f - \chi_r = -cs$ with $\dfrac{k_f}{k_r} = K_c^{\,s}$
$\delta_f - \delta_r = -ds$

33. Detailed balance is a consequence of microscopic reversibility.

Literature Cited

1. K. J. Laidler, *Chemical Kinetics*, Harper & Row, New York, NY, 1987. p. 300.
2. R. K. Boyd, "Some common oversimplifications in teaching chemical kinetics," *J. Chem. Ed.*, **1978**, *55(2)*, 84-9.
3. M. J. Pilling, I. W. M. Smith, "Modern Gas Kinetics: Theory, Experiment, and Application," Blackwell, Oxford, England, 1987. Section C2.2.
4. D. W. King, H. A. Lounsbury, F. J. Millero, "Rates and Mechanism of Fe(II) Oxidation at Nanomolar Total Iron Concentrations," *Environ. Sci. Technol.*, **1995**, *29*, 818-824.
5. T. S. Briggs, W. C. Rauscher, "An Oscillating Iodine Clock," *J. Chem. Ed.*, **1973**, *50(7)*, 496.
6. D. Kondepudi, I. Prigogine, *Modern Thermodynamics*, Wiley, Chichester, England, 1998, Sec. 19.4.
7. P. De Kepper, I. R. Epstein, "A Mechanistic Study of Oscillations and Bistability in the Briggs-Rauscher Reaction," *J. Am. Chem. Soc.*, **1982**, *104*, 49-55.
8. R. A. Alberty, "Principle of Detailed Balance in Kinetics," *J. Chem. Ed.*, **2004**, *81(8)*, 1206-9.
9. J. H. Harte, *Consider a Spherical Cow, A Course in Environmental Problem Solving*, University Science Books, Mill Valley, CA, 1988. Chapt. III.A.5

10. K. Brocklehurst, C. M. Topham, “In defense of the general validity of the Cha method of deriving rate equations. The importance of explicit recognition of the thermodynamic box in enzyme kinetics,” *Biochem. J.*, **1992**, *282*, 261-265.

11. K. Brocklehurst, C. M. Topham, “Some classical errors in the kinetic analysis of enzyme reactions,” *Biochem. J.*, **1993**, *295*, 897-902.

12. H. Strehlow, *Rapid Reactions in Solution*, VCH, Weinheim, FRG, 1992. Sec. 5.1.

13. D. Colquhoun, K. A. Dowsland, M. Beato, A. J. R. Plested, “How to Impose Microscopic Reversibility in Complex Reaction Mechanisms,” *Biophys. J.*, **2004**, *86*, 3510-18.

14. K. J. Laidler, *Chemical Kinetics*, Harper & Row, New York, NY, 1987. pp. 248, 294,300.

15. K. Denbigh, *The Principles of Chemical Equilibrium, 4th Ed.*, Cambridge University Press, Cambridge, Great Britain, 1981. Sec. 15.5.

16. C. A. Hollingsworth, “Equilibrium and the rate laws for forward and reverse reactions,” *J. Chem. Phys.*, **1952**, *20*, 921-2.

Further Reading

K. J. Laidler, *Chemical Kinetics*, Harper & Row, New York, NY, 1987.

J. W. Moore, R. G. Pearson, *Kinetics and Mechanism, 3rd Ed.*, Wiley, New York, NY, 1981.

M. R. Wright, *An introduction to Chemical Kinetics*, Wiley, Chichester, England, 2004.

M. Boudart, *Kinetics of Chemical Processes*, Butterworth-Heinemann, Boston, MA, 1991.

Problems: Kinetic Mechanisms

1. Consider the gas phase oxidation of HBr: $4\,HBr + O_2 \rightarrow 2\,H_2O + 2\,Br_2$. The following mechanism has been proposed:

$$HBr + O_2 \xrightarrow{k_1} HOOBr$$

$$HOOBr + HBr \xrightarrow{k_2} 2\,HOBr$$

$$HOBr + HBr \xrightarrow{k_3} H_2O + Br_2$$

Assume all unidirectional steps. Comment on the validity of this mechanism.

2. For the $H_2 + I_2$ reaction, $H_2 + I_2 \rightarrow 2\,HI$, the empirical rate law is

$$\upsilon = \frac{d[HI]}{dt} = k\,[H_2][I_2]$$

The empirical rate law matches the stoichiometry of the reaction. Why can’t you conclude that the mechanism is a simple single-step mechanism?

3. The gas phase decomposition of acetic acid at 1189 K proceeds by way of two parallel reactions:

(1) $CH_3COOH \rightarrow CH_4 + CO_2$ $k_1 = 3.74\ s^{-1}$
(2) $CH_3COOH \rightarrow H_2C{=}C{=}O$ $k_2 = 4.65\ s^{-1}$

What is the maximum ratio of $H_2C{=}C{=}O$ to CH_4 obtainable at this temperature?

4. The gas phase reaction, $2\ NO + 2\ H_2 \rightarrow N_2 + 2\ H_2O$, is known to have the rate law:

$$\upsilon = k\,[NO]^2[H_2]$$

Can this mechanism be a one-step mechanism? If not, suggest a possible two-step mechanism and suggest the rate determining step. You can use unidirectional elementary steps.

5. Use the finite difference approximation to integrate the rate law for a second order reaction, $A + B \rightarrow P$ with a rate constant of 0.05 $M^{-1}\ s^{-1}$. Choose the initial concentrations $[A]_o = 1.00$ M and $[B]_o = 0.50$ M. Integrate to at least 100 s. Use Excel for the integration. Compare to the exact expression and the results from the Web based "*Kinetics Mechanism Simulation*" applet or *MathCad* or *MatLab*.

6. Use the finite difference approximation to integrate the rate law for the two step mechanism:

$$A + B \xrightarrow{k_1} X \qquad\qquad X \xrightarrow{k_2} P$$

with rate constants $k_1 = 0.05\ M^{-1}\ s^{-1}$ and $k_2 = 0.2\ s^{-1}$. Choose the initial concentrations $[A]_o = 1.00$ and $[B]_o = 0.50$ M. Use $\Delta t = 1$ s for a maximum time of at least 50 s. Use Excel and compare to the results from the Web based "*Kinetics Mechanism Simulation*" applet or *MathCad* or *MatLab*. A useful comparison is to find the maximum concentration of the reactive intermediate.

7. The purpose of this exercise is to understand the statement: "the intrinsically slow step is the rate determining step." (a) Plot the integrated time course for the first-order mechanism:

$$A \xrightarrow{k_a} P$$

using $k_a = 0.05\ s^{-1}$ and $0.20\ s^{-1}$. Plot both [A] and [P] for each case. Use an initial concentration of $[A]_o = 1$. Use the Web based "*Kinetics Mechanism Simulation*" applet or *MathCad* or *MatLab*.
(b) Similarly find the integrated time course for the consecutive first-order mechanism:

$$A \xrightarrow{k_1} X \xrightarrow{k_2} P$$

Use $k_1 = 0.2\ s^{-1}$ and $k_2 = 0.05\ s^{-1}$. Predict the step that will be the rate determining step. Plot the time course for A, X, and P. To which curve in part (a) does the disappearance of A correspond, k=0.05 or k=0.20 s^{-1}? To which curve in part (a) does the appearance of product correspond? According to the plot, which step is the rate determining step? Is the intrinsically slow step the rate determining step?

8. Consider the reaction: $H_2O_2 + 2H^+ + 2\ I^- \rightarrow I_2 + 2\ H_2O$, with the proposed mechanism:

$H^+ + I^- \rightleftarrows HI$	rapid equilibrium
$HI + H_2O_2 \rightarrow H_2O + HOI$	slow
$HOI + I^- \rightarrow I_2 + OH^-$	fast
$OH^- + H^+ \rightarrow H_2O$	fast

Show that this mechanism is consistent with the experimental rate law:

$$\upsilon = k\,[H^+][I^-][H_2O_2]$$

9. A possible mechanism for 3rd order reactions is:

$$A + M \underset{k_{-1}}{\overset{k_1}{\rightleftarrows}} AM$$

$$AM + A \xrightarrow{k_2} A_2 + M$$

Show that the rate law can be expressed as: $\frac{d[A_2]}{dt} = \frac{k_1k_2[A]^2[M]}{k_{-1} + k_2[A]}$

10. Use the steady-state approximation to determine the rate law for the following mechanism:

$$A \underset{k_2}{\overset{k_1}{\rightleftarrows}} B \qquad\qquad B + C \xrightarrow{k_3} D$$

11. Determine the overall rate law for the proposed $H_2 + I_2$ mechanism:

$$I_2\,(g) \underset{k_{-1}}{\overset{k_1}{\rightleftarrows}} 2\,I\,(g)$$

$$H_2\,(g) + I\,(g) \xrightarrow{k_2} HI\,(g) + H\,(g)$$

$$H\,(g) + I_2\,(g) \xrightarrow{k_2'} HI\,(g) + I\,(g)$$

12. The decomposition of HI is given by $2\,HI \rightarrow H_2 + I_2$. One proposed mechanism is:

$$HI \xrightarrow{k_1} H + I$$

$$H + HI \xrightarrow{k_2} H_2 + I$$

$$2\,I + M \xrightarrow{k_3} I_2 + M$$

Use the steady-state approximation to find the rate law for this mechanism. Show that this mechanism does not agree with the experimentally determined rate law:

$$\frac{d[H_2]}{dt} = k\,[HI]^2$$

13. (a) Determine the integrated rate law for the Michaelis-Menten mechanism. Note that during the portion of the reaction where the steady-state approximation applies, – d[S]/dt = d[P]/dt so that Eq. 4.2.31 becomes:

$$-\frac{d[S]}{dt} = \frac{k_1\,[E]_o[S]}{(k_M + [S])}$$

(b) Show that for short times, [S] is a linear function of time: $[S] - [S]_o = -\frac{[E]_o([S]_o + 1)\,k_1}{k_M}\,t$

[Hint: you can approximate ln(x) ≈ x – 1, when x is close to 1.]

14. Use the "*Kinetic Mechanism Simulation*" applet to numerically integrate the rate laws for the Michaelis-Menten enzyme mechanism, Eq. 4.2.22. Set $k_2 = 0.40\ M^{-1}s^{-1}$, $k_{-1} = 0.1\ s^{-1}$, and $k_1 = 0.10\ s^{-1}$. Use the initial conditions $[S]_o = 1.0$ M and $[E]_o = 0.1$ M. Such a large enzyme

concentration will make the plot scaling more convenient. (a). Plot [S], [ES], and [P] for a maximum of 300 s to verify the linear time course for short times. (b). To observe the pre-induction lag, using the same conditions, plot [ES] and [P] for a maximum time of 15 s.

15. Consider the Lindemann-Henshelwood Mechanism for first-order reactions. Compare the net rate of the pre-equilibrium step to the rate of the unimolecular step during the majority of the time course of the reaction. Look at the rate dependence after any induction period.

16. The following mechanism has been proposed for an enzyme reaction with two substrates, A and B:

$$E + A \underset{k_{-1}}{\overset{k_1}{\rightleftarrows}} EA$$

$$EA + B \xrightarrow{k_2} EAB + Y$$

$$EAB \xrightarrow{k_3} E + P$$

where EA and EAB are enzyme substrate complexes. Assuming that k_2 and k_3 are large compared to k_1, show that the mechanism gives the rate law:

$$\frac{d[P]}{dt} = \frac{k_1\, k_2\, [E][A][B]}{k_{-1} + k_2\, [B]}$$

17. Consider the following proposed mechanism for the decomposition of ozone. M is an unreactive gas molecule that collides with the ozone to break the ozone apart:

$$O_3 + M \underset{k_{-2}}{\overset{k_2}{\rightleftarrows}} O_2 + O + M \qquad\qquad O + O_3 \xrightarrow{k'_2} 2\, O_2$$

Assume $k'_2 \gg k_2$. Show that the rate law that corresponds to this mechanism is:

$$\frac{d[O_2]}{dt} = \frac{3\, k'_2\, k_2\, [O_3]^2[M]}{k_{-2}[O_2][M] + k'_2[O_3]}$$

18. Report all six of the rate constants for the kinetics of proton exchange in aqueous solution of acetic acid, Figure 4.5.1. The reaction was studied at pH = 4.74 with the acetic acid and acetate concentrations both 0.100 M. The rate constants determined from temperature jump kinetics studies are $k_{31} = 4.5\text{x}10^{10}\ M^{-1}\ s^{-1}$ and $k_{23} = 1.8\text{x}10^{10}\ M^{-1}\ s^{-1}$.[1] Use the data from Example 3.6.1 for the auto-protolysis constants for water. The K_a for acetic acid is:

$$K_a = \frac{[H^+][OAc^-]}{[HOAc]} = 1.75\text{x}10^{-5}\ M$$

19. The following three-step mechanism has been proposed for the oxidation of HBr to Br_2:

$$HBr + O_2 \xrightarrow{k_1} HOOBr$$

$$HOOBr + HBr \xrightarrow{k_2} 2\, HOBr$$

$$HOBr + HBr \xrightarrow{k_3} H_2O + Br_2$$

To give the proper overall stoichiometry, this last step must be doubled. All the steps are unidirectional. Show that the corresponding rate law can be expressed as:

$$\frac{d[Br_2]}{dt} = k_1\,[HBr][O_2]$$

20. For some oscillating mechanisms, after a short initial period, the same cycle results no matter the starting conditions. For such cases the plot of the oscillating concentrations is called a **limit cycle**. Does the Lotka-Volterra mechanism give a limit cycle? [Hint: repeat Example 4.4.1 but with initial conditions $[A]_o = 1$ M and $[B]_o = 0.5$ M. Does the same cycle result as in Figure 4.4.2b?]

21. The Lotka-Volterra mechanism with all irreversible steps, Eqs. 4.4.6, are unrealistic in several ways. The result is that oscillations occur for too wide a range of rate constants and starting conditions. In addition, the system does not evolve towards a steady state. Modify the mechanism to include reversible reactions for the formation of A and B (steps 1 and 2), but leave the formation of products as irreversible. Run a simulation with the same conditions as in Example 4.4.1, except set the equilibrium constants for the formation of A and of B at 20. Comment on the results.

22. The "Brusselator" or "trimolecular" mechanism is a more realistic model for oscillating systems than the Lotka-Volterra mechanism. The Brusselator displays most of the complex phenomena associated with reactions far from equilibrium and was centrally important in the development of non-equilibrium thermodynamics. The mechanism is:[2]

$$M \xrightarrow{k_1} A$$

$$N + A \xrightarrow{k_2} B + P$$

$$2\,A + B \xrightarrow{k_3} 3\,A$$

$$A \xrightarrow{k_4} Q$$

where M and N are held constant by running the reaction in a flowing system. A convenient set of conditions for simulation is to set all the rate constants to 0.10, $[M]_o = 1.00$ M and $[N]_o = 3.00$ M. Run kinetics simulations using *MatLab*, *Mathematica*, or the "*Kinetic Mechanism Simulation*" applet for three sets of initial conditions: (a) $[A]_o = [B]_o = 1.00$ M; (b) $[A]_o = 1.00$ M, $[B]_o = 2.00$ M; and (c) $[A]_o = 1.00$ M, $[B]_o = 3.00$ M, which are the steady state concentrations. Run the simulation for 300 s. Because the concentrations change rapidly over the time interval, you will need to choose a large number of time steps to ensure numerical accuracy, choose 15000 time steps. Plot the concentrations of A and B.

23. The Brusselator mechanism is given in the previous problem. (a). Find relationships for the steady state concentrations of A and B in terms of the rate constants. (b). Find the steady state concentrations for the conditions given in the previous problem: that is, all the rate constants equal to 0.10, $[M]_o = 1.00$ M, and $[N]_o = 3.00$ M.

24. The Belousov-Zhabotinsky reaction is an oscillating reaction based on the oxidation of malonic acid with $KBrO_3$, which is catalyzed by Ce(IV):

$$3\ CH_2(CO_2H)_2 + 4\ BrO_3^- \rightarrow 4\ Br^- + 9\ CO_2 + 6\ H_2O$$

The BZ reaction played a central role in the development of techniques to study oscillating reactions and in the theory of non-equilibrium thermodynamics.[2,3] The initiation step is the generation of $HBrO_2$, a key reactive intermediate, from BrO_3^-:

$BrO_3^- + Br^- + 2H^+ \rightarrow HBrO_2 + HOBr$ 1

The bulk of the $HBrO_2$ is produced auto-catalytically:

$BrO_3^- + HBrO_2 + 2\ Ce^{3+} + 3\ H^+ \rightarrow 2\ HBrO_2 + 2\ Ce^{4+} + H_2O$ 2

The intermediate $HBrO_2$ is consumed in the reactions:

$HBrO_2 + Br^- + H^+ \rightarrow 2\ HOBr$ 3

$2\ HBrO_2 \rightarrow BrO_3^- + HBrO + H^+$ 4

The oxidation of malonic acid is complex, but a simplified version includes first the bromination of malonic acid:

$HOBr + Br^- + H^+ \rightarrow Br_2 + H_2O$ 5

$Br_2 + CH_2(CO_2H)_2 \rightarrow BrCH(CO_2H)_2 + H^+ + Br^-$ 6

and the oxidation of malonic acid and bromomalonic acid by Ce^{4+}:

$Ce^{4+} + \frac{1}{2}\ [CH_2(CO_2H)_2 + BrCH(CO_2H)_2] \rightarrow \frac{1}{2}\ Br^- + Ce^{3+} + \text{products}$ 7

The products include CO_2, H_2O, and a mixture of organic acids. For modeling purposes, the oxidation of the malonic acid by Ce(IV) is represented by the net Ce(IV) to Br^- stoichiometry using:

$CH_2(CO_2H)_2 + Ce^{4+} + HOBr \rightarrow Ce^{3+} + Br^- + H^+ + \text{products}$ net 5-7

Field, Körös, and Noyes have developed a mechanism for the reaction that displays oscillations. With steps numbered according to the mechanistic steps listed above, the FKN mechanism is:[2,3]

$B + M \xrightarrow{k_1} A + P$ $k_1 = 1.28\ mol\ L\ s^{-1}$ 1

$A + M \xrightarrow{k_2} 2\ A + 2\ C$ $k_2 = 8.0\ mol^{-1}\ L\ s^{-1}$ 2

$A + B \xrightarrow{k_3} 2\ Q$ $k_3 = 8.0x10^5\ mol^{-1}\ L\ s^{-1}$ 3

$2\ A \xrightarrow{k_4} Q + M$ $k_4 = 2.0x10^3\ mol^{-1} L\ s^{-1}$ 4

$C + N \xrightarrow{k_5} B$ $k_5 = 1.0\ mol^{-1}\ L\ s^{-1}$ net 5-7

with $A = HBrO_2$, $B = Br^-$, $C = Ce^{4+}$, $M = BrO_3^-$, N = malonic acid, Q = HOBr

The H^+ and Ce^{3+} concentrations are roughly constant and are included through pseudo-rate constants. M and N are held constant by a flowing reactor. This mechanism is also called the "Oregonator." Do a numerical simulation of the FKN mechanism using *MatLab*, *Mathematica*, or the "*Kinetics Mechanism Simulation*" applet on the companion CD or the textbook Web site with the following conditions:

$[M] = [BrO_3^-] = 0.06\ M$, $[N] = [\text{malonic acid}] = 0.02\ M$
$[A]_o = [HBrO_2] = 2.0\text{x}10^{-7} M$, $[B]_o = [Br^-] = 2.0\text{x}10^{-5}\ M$, $[C]_o = [Ce^{4+}] = 1.0\text{x}10^{-4}\ M$.

Run the simulation for 750 s. Because the rate constants span almost six orders of magnitude, you will need to choose a large number of time steps to ensure numerical accuracy, choose 7500 or more total time steps. Plot the concentrations of A and B.

25. Consider a reaction $A \rightleftarrows B$ at equilibrium that can occur via a catalyzed path and an uncatalyzed path, with C the catalyst:

$$A + C \underset{k_{BC}}{\overset{k_{AC}}{\rightleftarrows}} B + C \qquad \text{and} \qquad A \underset{k_B}{\overset{k_A}{\rightleftarrows}} B$$

catalyzed *uncatalyzed*

Show that if 10% of the forward process at equilibrium occurs by the uncatalyzed path that 10% of the reverse process will also occur by the uncatalyzed path.

26. A random bi-substrate enzyme mechanism requires two substrates, but the substrates can bind to the enzyme in either order. One example is an enzyme that phosphorylates a protein using ATP as the phosphate source; ATP and the protein target are the two substrates. The mechanistic steps are:

$$E + A \rightleftarrows [EA]$$
$$E + B \rightleftarrows [EB]$$
$$[EA] + B \rightleftarrows [EAB]$$
$$[EB] + A \rightleftarrows [EAB]$$

all of which are assumed to be in quasi-equilibrium (in the same sense as the pre-equilibrium mechanism). The production of product is assumed to be essentially irreversible:

$$[EAB] \rightarrow E + \text{products}$$

Draw the quasi-equilibrium mechanistic steps as a four-state cyclic process, and give the relationship among the corresponding rate constants.

27. Consider the bidirectional reaction:

$$A \rightleftarrows B \rightleftarrows C$$

The initial rate law for the reaction, starting with A only, is experimentally determined to be:

$$\frac{d[A]}{dt} = -\,k_{obs,f}\,[A]_o$$

If the reverse reaction is run starting with C only, the initial rate law is determined to be:

$$\frac{d[C]}{dt} = -\, k_{obs,r}\, [C]_o$$

Why <u>isn’t</u> the equilibrium constant for the overall reaction given by the ratio of the initial forward to the initial reverse rate constants, $k_{obs,f}/k_{obs,r}$?

28. This problem is an example of finding the relationship among the rate constants in a mechanism and the overall equilibrium constant, based on detailed balance. For the mechanism:

$$A \;\underset{k_{-1}}{\overset{k_1}{\rightleftarrows}}\; B \;\underset{k_{-2}}{\overset{k_2}{\rightleftarrows}}\; C$$

determine the rate law for the overall forward process, υ_+, and the rate law for the overall reverse process, υ_-, at equilibrium. Show that the ratio of the resulting rate constants gives the overall equilibrium constant.

29. Using the Principle of Detailed Balance, show that the following mechanism generates the expected overall equilibrium ratio when the reaction is at equilibrium:

$$HI \underset{k_{-1}}{\overset{k_1}{\rightleftarrows}} H + I$$

$$H + HI \underset{k_{-2}}{\overset{k_2}{\rightleftarrows}} H_2 + I$$

$$2\, I + M \underset{k_{-3}}{\overset{k_3}{\rightleftarrows}} I_2 + M$$

overall:

$$K_{eq} = \left(\frac{[H_2][I_2]}{[HI]^2}\right)_{eq}$$

Derive the relationship between the rate constants and the overall equilibrium constant.

30. The reaction $A + B \rightleftarrows D$ is proposed to have the following mechanism:

$$A + B \underset{k_{-1}}{\overset{k_1}{\rightleftarrows}} C$$

$$C + M \underset{k_{-2}}{\overset{k_2}{\rightleftarrows}} D + M$$

where C is a reactive intermediate and M is an inert gas in large concentration. (a). Show that with appropriate approximations that the rate law is:

$$\frac{d[D]}{dt} = \left(\frac{k_1 k_2 [A][B][M] - k_{-1} k_{-2}\, [D][M]}{k_{-1} + k_2[M]}\right)$$

(b). In terms of the <u>overall</u> process, $A + B \rightarrow D$, near equilibrium, the <u>overall</u> rate law in terms of the initial reactant and the final product is:

$$\upsilon = \frac{d[D]}{dt} = k_f\, [A][B] - k_r\, [D]$$

Show that the equation in part (a) reduces to this last overall equation and find the relationship between the overall equilibrium constant and the four rate constants for the mechanistic steps from <u>this</u> equation. Discuss any approximations that you make for parts (a) and (b).

31. The reaction 2 A $\rightleftarrows$ C is proposed to have the following mechanism:

$$A \underset{k_{-1}}{\overset{k_1}{\rightleftarrows}} B$$

$$A + B \underset{k_{-2}}{\overset{k_2}{\rightleftarrows}} C$$

(a). Show that near equilibrium:

$$-\frac{1}{2}\frac{d[A]}{dt} = \frac{d[C]}{dt} = k_1[A] - \left(\frac{k_1 k_{-1}[A] + k_{-1}k_{-2}\,[C]}{k_{-1} + k_2[A]}\right)$$

(b). In terms of the <u>overall</u> process, 2A → C, <u>at equilibrium</u> we would write the <u>overall</u> rate law in terms of the initial reactant and the final product:

$$\upsilon = -\frac{1}{2}\frac{d[A]}{dt} = k_f\,[A]^2 - k_r\,[C]$$

Show that the equation in part (a) reduces to this last overall equation and find the relationship between the overall equilibrium constant and the four rate constants for the mechanistic steps.

32. For the reaction:

$$I_2 \underset{k_{-1}}{\overset{k_1}{\rightleftarrows}} 2\,I \qquad\qquad H_2 + 2\,I \xrightarrow{k_3} 2\,HI$$

find the relationship of the experimentally determined, overall activation energy to the activation energies for the individual mechanistic steps. Assume the rate law is:

$$\frac{d[HI]}{dt} = \frac{k_1\,k_3}{k_{-1}}\,[H_2][I_2]$$

33. Consider the S_N1 mechanism as a typical multi-step mechanism. For the case given in Eq. 4.2.17, the rate law has the form:

$$\frac{d[P]}{dt} = k\,\frac{[RX]\,[Nuc{:}^-]}{[X^-]} \qquad\qquad (k_{-1}\,[X^-] >> k_2[Nuc{:}^-])$$

with the effective rate constant, $k = k_2\,k_1/k_{-1}$. From Eq. 3.5.1, for the overall reaction with activation energy E_a: $k = A\,e^{-E_a/RT}$. This form of the Arrhenius equation can also be applied to each individual elementary step. For the individual elementary steps, for step 1: $k_1 = A_1\,e^{-E_{a1}/RT}$, for the reverse of step 1: $k_{-1} = A_{-1}\,e^{-E_{a-1}/RT}$, and for the formation of product: $k_2 = A_2\,e^{-E_{a2}/RT}$. Using these Arrhenius expressions, find the relationship between the overall pre-exponential factor and activation energy and the pre-exponential factors and activation energies for the individual elementary steps.

34. The connection between detailed balance and forward and reverse reaction rates for a multi-step mechanism is illustrated in Eqs. 4.5.3-4.5.10. The rate laws are written for each individual step in the mechanism, rather than the rate of appearance or disappearance of a particular species. Symbolize the net rates of the individual mechanistic steps as υ_1, υ_2, and υ_3:

$$\upsilon_1 = \frac{1}{V}\frac{d\xi_1}{dt} \qquad\qquad \upsilon_2 = \frac{1}{V}\frac{d\xi_2}{dt} \quad \text{and} \quad \upsilon_3 = \frac{1}{V}\frac{d\xi_3}{dt}$$

(a). Write the rate laws for the appearance of intermediate A_2, the appearance of intermediate X, and the appearance of product D in terms of υ_1, υ_2, and υ_3. The purpose is to show the relationship between the species specific and step specific methods of writing rate laws.
(b). For a multi-step mechanism at steady state, the rates of the individual steps are equal, Sec. 4.2. Use the results of part (a) for intermediates A_2 and X to prove this statement for this example mechanism. [Hint: apply the steady-state approximation].

35. Consider the S_N1 mechanism, Eqs. 4.2.6-4.2.7. The first mechanistic step is reversible and the second is uni-directional. Symbolize the net rates of the two mechanistic steps as υ_1 and υ_2:

$$\upsilon_1 = \frac{1}{V}\frac{d\xi_1}{dt} \quad \text{and} \quad \upsilon_2 = \frac{1}{V}\frac{d\xi_2}{dt}$$

(a). Write the rate law for the appearance of intermediate R^+ and the rate law for the appearance of product R-Nuc in terms of υ_1 and υ_2. The purpose is to show the relationship between the species specific and step specific methods of writing rate laws.
(b). For a multi-step mechanism at steady state, the rates of the individual steps are equal. Use the result of part (a) for intermediate R^+ to prove this statement for this example mechanism. [Hint: apply the steady-state approximation].

36. After a perturbation, the three unidirectional reactions in Eq. 4.5.1 approach a steady state very differently than a reversible process approaches equilibrium; the approach of the unidirectional steps to a steady state shows oscillatory behavior, but a reversible system approaches equilibrium in an exponential process.[4,5] (a) Use the Web based "*Kinetic Mechanism Simulation*" applet or *MatLab*, *Maple*, or *Mathematica* to numerically integrate the rate laws for the mechanism in Eq. 4.5.1. Set $k_{AB} = 0.1$, $k_{BC} = k_{CA} = 0.05$, and $[A]_o = 1$ while $[B]_o = [C]_o = 0$. Plot the approach to the steady state and verify the appearance of oscillations. (b) Change the rate law to match Eq. 4.5.2. Set the forward rate constants as in part (a) and the reverse rate constants to give the equilibrium constant for $A \rightleftarrows B$ as 2 and for $B \rightleftarrows C$ as 2. Compare the approach to equilibrium with part (a).

Literature Cited

1. H. Strehlow, *Rapid Reactions in Solution*, VCH, Weinheim, FRG, 1992. Sec. 5.1.
2. D. Kondepudi, I. Prigogine, *Modern Thermodynamics*, Wiley, Chichester, England, 1998, Sec. 19.4, Appendix 19.1.
3. R. J. Field, E. Körös, R. M. Noyes, "Oscillations in chemical systems II," *J. Am. Chem. Soc.*, **1972**, *94(25)*, 8649-8664.
4. R. A. Alberty, "Principle of Detailed Balance in Kinetics," *J. Chem. Ed.*, **2004**, *81(8)*, 1206-9.
5. J. H. Harte, *Consider a Spherical Cow, A Course in Environmental Problem Solving*, University Science Books, Mill Valley, CA, 1988. Chapt. III.A.5.