

MA 262	Vector Calculus	Spring 2023
	MWF 1 - 1:50	Mudd 405

Professor: Scott Taylor

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Prerequisites: MA 160 or permission of instructor

Text: Colley, *Vector Calculus* 4/e, Pearson.

Office Hours in Davis 217: (subject to change)

Tuesday: 10:30 - 12

Wednesday: 11:30 - 1

Thursday: 1 - 2:30

and by appointment!

TA Hours: 8 - 10 PM on Wednesday evenings.

The Course: MA 262, building off the content of MA 160, develops the calculus of functions in all dimensions. The main goal of the course is to relate the concepts of “derivative” and “integral” in all dimensions by generalizing the Fundamental Theorem of Calculus. Along the way you will become adept at using the language and notation of vectors to express key ideas in geometry and physics.

Objectives for increasing mathematical maturity:

- Develop an intuition for higher dimensions by comparing high dimensional geometry and calculus to low dimensional geometry and calculus.
- Use vectors and vector fields to solve problems in geometry and physics.
- Use geometric and physical intuition to understand mathematical objects.
- Increase understanding of the relationship between definitions, examples, theorems, and proofs in mathematics.
- Begin to explore the intrinsic nature of many geometric definitions (eg. curvature).
- Begin to explore the relationship between geometry and topology by discussing notions such as orientation and path-independence.
- Understand the relationship between Introductory Calculus and Vector Calculus
- Engage in significant self-teaching of mathematics.
- Effectively communicate mathematics.

Major Course Content Objectives

- Understand the meaning of multivariable integrals and the connection to the change of variable theorem.
- Be able to set up and compute multivariable integrals, using Fubini’s theorem, software, and the change of variable theorem.
- Use linear algebra to understand the definition and behaviour of the derivatives of functions $f: \mathbb{R}^n \rightarrow \mathbb{R}^m$.

- Use derivatives to understand intrinsic properties of curves in \mathbb{R}^2 and \mathbb{R}^3 . These include arc length, curvature, and unit tangent vectors.
- Use the calculus of space curves to model various physical situations
- Understand the mathematical and physical significance of the gradient, divergence, and curl of a vector field.
- Understand the mathematical and physical significance of path integrals of scalar and vector fields.
- Understand the mathematical and physical significance of surface integrals of scalar and vector fields.
- Understand the statements of Green's, Gauss', and Stokes' theorems.
- Understand the main ideas in the proof of Green's theorem.
- Understand the concepts of conservative vector fields and their potential functions.

Attendance: I value your involvement in the class, therefore class attendance is mandatory. Absence for official Colby activities requires prior approval. Absence for religious reasons will be considered excused if the policy in the catalogue is followed. I reserve the right to take attendance. More than 3 recorded unexcused absences will result in the reduction by 1/3 of the final course grade. Excessive tardiness or early departure may also result in such a reduction.

Please exercise discretion in matters of health. If you have communicable respiratory disease symptoms, such as those for a cold or Covid, please wear a mask or stay home from class, as appropriate.

Computing Resources:

You will be required to use software to complete homework and projects throughout the semester. Please see the separate page on your options for doing so.

Academic Honesty: Academic honesty means that the work you present is your own, the ideas you communicate actually represent what you think and feel, and that you are upfront about your sources and inspirations. An act of academic dishonesty can either be intentional or unintentional. In either case, there are both informal and formal consequences. Being scrupulously honest is essential for the functioning of the college, for getting the most out of your own education, and for the success of mathematics as a discipline.

In this course, the greatest temptation towards academic dishonesty is presenting someone else's work as your own (perhaps with minor modifications). If someone other than the TA or professor gives you the key idea for a problem, you must credit them in your write-up. This includes any help you receive from online sources and applies even if you received help from a student who received help from a TA or the professor. Acknowledging a source will never result in penalty. You are encouraged to work together and share ideas, but you may not copy another person's proof. Furthermore, you may not share LaTeX source code with other students and, in your final project, you must give complete citations for any ideas, phrasings, or images you get from another source. The solutions manual for the course text is (illegally) available online. If you consult it, you must also give credit to the solutions manual.

To assist with this, **every assignment you turn in must be accompanied by a standard cover sheet**, downloadable from the course webpage.

Academic dishonesty is a serious offense against the college. Sanctions for academic dishonesty are assigned by an academic review board and may include failure on the assignment, failure in the course, or suspension or expulsion from the College.

TA resources: There are TAs assigned to MA 262. They will hold some evening help hours. The purpose of these help hours is to help connect you with other students, point you to helpful resources in the book and notes, and ask questions to help you figure out “the next step” for a problem you are stuck on. They are not to just tell you the next step or to reteach you material from earlier calculus courses, though of course they can provide quick helpful reminders. For TA resources to be effective you need to have worked on the problems for a *significant* length of time

Learning Differences: Students with learning differences are encouraged to meet with me to discuss strategies for success. I am committed to helping all students succeed and to making reasonable accommodations for documented learning differences.

Evaluation: The numerical course grade will be a weighted average of the cumulative grades with weightings as follows:

25%	Homework
15%	Quizzes and reading assignments
15%	minimum of Exam 1 and Exam 2
20%	maximum of Exam 1 and Exam 2
20%	Final Exam
5%	Final Project

Caveat: earning less than 50% on each of the three exams will result in a grade of “F” for the course.

Course letter grades will be assigned (subject to the above caveat) according to the following scale. Any curve will be determined at the end of the course according to the discretion of the instructor. An A+ may be awarded to students who do exceptionally well and who demonstrate an exceptional interest in the course.

93–100 %	A	90–93 %	A-	87–90 %	B+	83 – 87%	B
80– 83 %	B-	77–80 %	C+	73–77 %	C	70 – 73%	C-
67– 70 %	D+	63–67 %	D	60–63 %	D-	below 60%	F

Homework: Homework is the most important part of this course – it’s when you get to put into practice the concepts you’ve played with during class. Some of the homework questions may require you to explore some topic which we didn’t discuss in class. The purpose of such questions is to help you develop the ability to read and learn mathematics on your own. If you go into a mathematical or scientific career, there will undoubtedly be times when you need to teach yourself some mathematics. If, however, all the homework problems fall into this category, you should check to make sure that you are working on the correct assignment. Some problems will be listed as suggested. You are encouraged to try them, but need not turn in your solutions.

Weekly homework will generally be due on **Fridays** and will always be posted to the course webpage. You are responsible for checking the webpage. If no homework assignment is posted, you should refresh the webpage on your browser and, if that doesn’t work, email me to let me know. In the special circumstance that there is no new homework, the webpage will make note of that. If you will not be in class on the day that homework is due you should arrange to turn it in at my office or to have a friend bring it to class. Late homework may be penalized. Students who are on-campus and not in isolation or quarantine should turn in physical homework.

You should start the homework early, some problems will require multiple attempts and careful thought. If you are having substantial difficulty with a particular problem or the entire homework set you should email me or come to office hours. I am eager to help you!

You are encouraged to work with a partner on the homework, but all work should be your own. In other words, you may discuss particular problems but you may not copy someone else's solution. Doing so violates academic honesty. As the course progresses, the question of how much work to show will arise. I encourage you to use common sense. If the work pertains to concepts discussed in class or in the reading, you should show it. If the work requires substantial effort and thought, you should show it. If the work is simply evaluating an integral using methods learned in MA 125, you do not need to show it. If the work is simply elementary algebra, you do not need to show it (although showing it may help the grader follow your work). You may even use software to evaluate the 1-variable integrals or derivatives that arise.

In general, your work is your answer. It is possible for someone to obtain a correct answer but to not receive full credit because their work is incorrect. Conversely, (almost entirely) correct work with an incorrect answer may receive full credit.

Homework must be very neat. This means: no messy scratchwork, no cramped writing, no huge eraser marks. Multiple pages should be stapled and the problems should be in order with section and problem number clearly indicated. If these guidelines are not followed you may be penalized. If you are incapable of writing neatly, you should type your solutions. \LaTeX is the most popular mathematical typesetting software. You may not use Microsoft Word, Pages, or other word-processor.

You will occasionally have a reading assignment or calculus review assignment due in class on a day other than the regular homework due date. These assignments will require that you do a certain amount of reading and answer some questions. Usually the reading will cover material we have not yet or will not discuss in class. I will always assume that you have done the reading by the time that it is due and will not regurgitate in class material you have read on your own. The review exercises will ask you to review material typically covered in MA 125 or MA 160. These will usually be due the class period after I assign them and are in addition to any other homework. It may be that a review exercise covers material you have not actually been taught before. In that case you are expected to either teach it to yourself or to see me for help.

Late assignments: If you have a **very good** reason, you may turn in a homework assignment up to one week late. It is expected that you will not make use of this more than once or twice during the semester. Whether an assignment is a few hours or a few days late, you are required to have an electronic backup (e.g. a scan) of it in case it gets misplaced. Also, I am under no obligation to return late homework in a timely way.

Quizzes: You will be given a quiz every so often, with at least two days warning. If you are absent on the day a quiz is given, it may be made up within one week of when it was given. You are responsible for requesting a make-up quiz.

Exams: There will be two in-class exams and a final exam. Each exam is cumulative, although the final exam is "more cumulative". Exams will be designed to test your understanding of the course material, not just your computational abilities. Indeed, you will not be asked to compute any integrals. You must understand, and communicate your understanding of the material. Computers, electronic devices, phones, textbooks, notes, and other people may not be used on the exam. The in-class exams will be given on **March 8** and **April 19**. The final exam is TBD. It may not be rescheduled for personal convenience (including airline reservations).

How to succeed in this course:

Vector Calculus is an intense 200-level mathematics course. The course, therefore, expects a level of sophistication and mathematical maturity beyond that expected by introductory calculus courses. In particular: you are responsible for your own learning and the professor is responsible for assisting you in your quest. In order to truly understand vector calculus you need to be willing to do computations and you need to be willing to wrestle with ideas. The following advice is meant to provide suggestions on get more out of the class and to improve your grades:

- (1) *Do some math everyday.* Really.

Learning mathematics is a lot like learning to play an instrument, play a sport, or learn a language. You must practice everyday. The daily homework problems are intended to help you in your daily practice. But you should spend additional time each day working on the weekly homework, reading the text, and studying previous material. On average, you should spend 2 - 3 hours studying for each hour spent in class. In our case, that's 6-9 hours per week of homework and studying.

- (2) *Participate in class.* Yes, you.

Asking and answering questions is a great way to stay engaged with the material and verify for yourself that you know what's going on. The more people that participate, the more fun class is. I, and the rest of the class, value your questions and your answers, right or wrong. In fact, giving a wrong answer to a questions is a great way to learn the right answer. Make an effort to connect each class's activities to previous classes. Try to predict where the material will be going in the future. Take good notes and listen. If you can't do both, make a deal with a buddy: you take notes and they listen one day and the next day you switch.

- (3) *Read the textbook.*

The lectures and the textbook will often present slightly different views on the same material. The examples in the text will (for the most part) differ from the example presented in class. As you read the text, relate what the text does to what happened in class: where is it the same, where is it different? If it differs, which approach do you like better— what are the pros and cons of each?

- (4) *Form a study group.*

Introduce yourself to other people in the class and meet up outside of class to study and work on homework. Be sure that you don't copy answers, but learn from each other and then write the answers on your own. Compare lecture notes to be sure you copied everything correctly. Ask each other questions and explain material to each other. Calculus-After-Hours is for 100-level calculus students, but it makes the math department a happening place – it's a great place to meet up with your classmates.

- (5) *Write your own problems.*

As you study for the exams, look back at all the examples done in class, in the text, or on homework. Try modifying them to make up your own problems. Try to solve your own problems: what makes your problems easier or more difficult than the ones you've seen before? Feel free to show me what you've done.

- (6) *Be curious.* We will be spending a lot of time, exploring the theory behind Vector Calculus. Try to be curious about why Calculus works. Ask “why” a lot. Feel free to ask about why we're learning something, but also ask about why the ideas came into existence and why

the math works one way, but not another. Schedule time for curiosity. Block off 2 hours (most) every day for Calculus. Any time that's not spent working on homework or studying, use for playing with the ideas. Use a computer to sketch graphs of vector fields and curves – see if you can predict they way they'll look. Coming up with your own questions is a great way to learn to care about the underlying mechanics of Calculus. Computers can compute; humans ask creative questions.

(7) *Visit me in my office.*

I love working with students and I love to help you understand and appreciate the beautiful world of mathematics. Ask crazy questions about the course. Ask questions about my research. Tell me about your past math experiences. Tell me about what subjects you love. Let me know when you have a concert or athletic event.

(8) *Spread the studying out over the semester.*

If you do math everyday, as suggested above, you won't have to cram for exams. You'll be able to sleep and, consequently, to think. You'll be happier and more relaxed. You'll have time to write papers for your other classes. You'll have time to appreciate the snow and the spring. You don't need to pull all-nighters.

(9) *Have an exam strategy.*

For the in-class exams, you will have only 50 minutes. Be prepared to do some problems very rapidly and be prepared to think about others. If an example was done in class or on homework, you should be able to repeat it very quickly on the exam. Know what you find difficult and what you find easy. Do the easy things first and then the difficult things. Write something for every problem. If you get stuck, tell me how you'd solve it if you could get unstuck. Figure out what the problem is testing and tell me what you know about that area. If the problem is too hard, rewrite it to make it easier. I love to give partial credit. Give me a reason to give you some. If you find yourself getting nervous: breathe deeply, remind yourself you've studied thoroughly, then figure out how to do the problem. Keep an eye on the time and don't spend too long on any one problem.