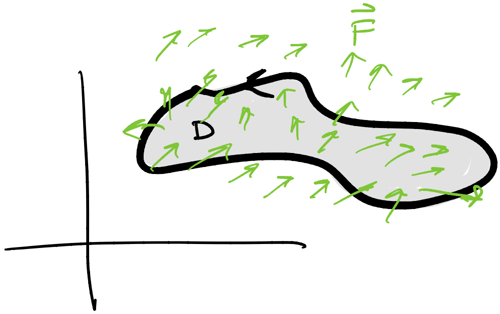
# Green Theorem: SUMMARY OF TRICKS

Assume $D$ is compact, $\partial D$ piecewise $C^1$, oriented so $D$ on left

$\vec{F}$ $C^1$ vector field defined on an open set containing $D$ then




$$\int_{\partial D} \vec{F} \cdot d\vec{s} = \iint_D \operatorname{scurl} \vec{F} \, dA$$

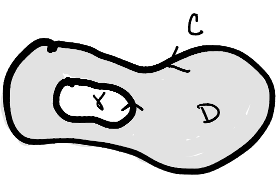
① If $\operatorname{scurl} \vec{F} = 1$ then

$$\text{Area}(D) = \iint_D \operatorname{scurl} \vec{F} \, dA \overset{\text{Green}}{=} \int_{\partial D} \vec{F} \cdot d\vec{s}$$

For ex $\vec{F}(x,y) = \begin{pmatrix} -y/2 \\ x/2 \end{pmatrix}$ or $\vec{F}(x,y) = \begin{pmatrix} 0 \\ -x \end{pmatrix}$ or $\vec{F}(x,y) = \begin{pmatrix} y \\ 0 \end{pmatrix}$

"Can calculate area by integrating along boundary"

② If $\operatorname{scurl} \vec{F} = 0$ and $\partial D$ has two components $C, \gamma$ both oriented counterclockwise




$$\int_C \vec{F} \cdot d\vec{s} - \int_\gamma \vec{F} \cdot d\vec{s} = \int_{\partial D} \vec{F} \cdot d\vec{s}$$

$$\overset{\text{Green}}{=} \iint_D \operatorname{scurl} \vec{F} \, dA$$

$$= 0$$

So $\int_C \vec{F} \cdot d\vec{s} = \int_\gamma \vec{F} \cdot d\vec{s}$

"If two curves co-bound a region and if they have the same orientation and if $\operatorname{scurl} \vec{F} = 0$ on the region, then the integral of $\vec{F}$ over one curve equals the integral over the other curve"