MA 262 Vector Calculus Spring 2023

Exam 2 Study Guide

The in-person exam will be given during class. It will consist of approximately 10 questions. On average each question will take a very well-prepared student about 5 minutes (i.e. like a quiz question).

In addition to this study guide, you should study your course notes, quizzes, review sheets, and homework assignments. With regard to the homework, keep in mind that some problems are simply not suitable for an exam, while others could be adapted to be an exam problem. Additionally, some problems are extremely difficult and time-consuming the first time you encounter them, but quick and much easier the second time you encounter them. Some problems, on the other hand, are difficult and time-consuming even the second time you do them. Such problems do not make good exam questions.

The general topics covered by the exam are listed below. For each, several open-ended questions are listed. Use these questions to make your own study guide for the more conceptual parts of the course.

You may not use any books, notes, calculator, internet sources or other people. Be prepared to explain facts and how they are related to each other. If we went over a proof in class, you should be prepared to discuss it on the exam.

Conceptual:

(1) Vector Field Line Integrals: What do they measure and why? What is circulation? What is flux? What are some ways to tell that a line integral will be zero, without actually calculating it? What are some ways to tell it will be non-zero? How do you show the line integral is intrinsic to oriented curves? Why are the following three statements equivalent: 1. A vector field has path independent line integrals; 2. The integral of the vector field over every closed loop is zero; 3. The integral of the vector field over every embedded closed loop is zero?

(2) Conservative Vector Fields: What is the definition? What is the Fundamental Theorem of Calculus for Conservative Vector Fields and how do you prove it? What is the connection between finding a potential function and solving partial differential equations? What can you say about the flow lines of a conservative vector field? What can you say about the integral of a conservative vector field over a closed loop? What does it mean for a vector field to have path independent line integrals? Why does a conservative vector field have path independent line integrals? Why are all vector fields with path independent line integrals conservative? Why are all vector fields having the property that "integrals over closed loops are zero" conservative?

(3) Curl and Divergence: What are the partial derivative formulas for curl and divergence? What are the integral formulas for curl and divergence? What do curl and divergence measure; how do you know? Why is the curl of a conservative vector field equal to zero? Why is the divergence of curl of a vector field equal to zero? What is an example of a non-conservative vector field whose (scalar) curl is zero? How can we obtain the partial derivative formula for scalar curl from the integral formula?

(4) Green's Theorem: What does Green's theorem say (include both the hypotheses and the conclusion)? What are the main ideas in the proof of Green's theorem? How do we use Green's Theorem to show that if a 2-dimensional vector field has scalar curl equal to 0 on a simply connected domain, then it is conservative there?

Computations: You should be able to compute the following. You do not need to actually solve any integrals, but you need to get them to a point where they would make sense to a strong Calc 1 student.

(1) The superposition of two vector fields or potential functions, especially those arising from the gravitational or electromagnetic fields.

(2) The line integral of a vector field over a curve

(3) The circulation of a vector field around a circle, triangle, square, or polar rectangle, using shortcuts as necessary to avoid actually needing parameterizations.

(4) The flux of a vector field across a circle, triangle, square, or polar rectangle, using shortcuts as necessary to avoid actually needing parameterizations.

(5) The scalar curl of a 2-dimensional vector field using both the partial derivative and the integral formula, using shortcuts as necessary to avoid actually needing parameterizations.

(6) The divergence of a 2-dimensional vector field using both the partial derivative and the integral formula, using shortcuts as necessary to avoid actually needing parameterizations.

(7) For a given region D in $\mathbb{R}^2$ compute the integrals on both sides of the equation in the conclusion of Green's theorem and verify that you get the same answer.

Practice Problems: These are offered as additional practice, use them to focus on areas where you feel you need more practice. You do not need to work all of them. On the exam, be aware that some problems can be done in multiple ways with one way taking significantly less time than another. Before working any problem, consider what the best approach might be.

(1) Revisit all old homework and quiz problems and work them again. (Don't just look over the problems and solutions)
(2) Problems 1 - 12 on page 235. Which of those calculations can be done with both partial derivative and integral versions of divergence or curl?
(3) Problems 39, 43, 44 on page 242-243.
(4) Problems 1 - 33, on page 427.
(5) Problems 1 - 10, on pages 436 - 437. (Note that the book puts a circle on an integral to indicate the curve being integrated over is a closed loop.)
(6) Problems 1 - 28, on pages 448 - 449.
(7) The true-false questions on page 450.