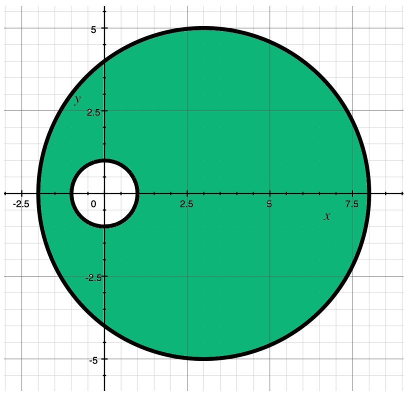
MA 262: Exam 2 Redo **Name:**________________________

You may not use textbooks, notes, electronic devices or refer to other people (except the instructor). You may not have a phone visible during the exam and you may not leave the room until you are read Show all of your work; **your work is your answer**.

Problem	Score	Possible
1		20
2		20
3		20
4		5
5		20
6		5 (bonus)
Total		85

Remember to thoroughly explain each answer. **Your work is your answer.**

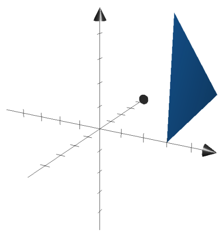
Problem 1: Consider the region S shown below. It is the ring between the circles with equations $x^2+y^2=1$ and $(x-3)^2+y^2=25$. Let C_1 be the inner circle and C_2 be the outer circle. Orient both C_1 and C_2 counter-clockwise.



(1) Suppose that $\mathbf{F}$ is a vector field defined on S such that $\operatorname{scurl}\mathbf{F}=2$. If $\int_{C_1} \mathbf{F}\cdot d\mathbf{s} = 6\pi$, what is $\int_{C_2} \mathbf{F}\cdot d\mathbf{s}$?

(2) Suppose that $\mathbf{G}$ is a vector field defined on S such that $\operatorname{div}\mathbf{G}=6$. If the flux of $\mathbf{F}$ across C_1 is equal to 9, what is the flux of $\mathbf{F}$ across C_2?

Problem 2: Let $\mathbf{F}(x,y,z) = \begin{pmatrix} 5x^2 \\ z \\ 2y \end{pmatrix}$. Let S be the triangle in $\mathbb{R}^3$ with vertices $(3,0,0)$, $(3,0,5)$, and $(3,7,0)$. Give S the orientation so that its normal vector is $(-1,0,0)$ at each point. See the image below.



(1) Find a parameterization of ∂S; you may provide separate parameterizations for each edge if you like.

(2) Does your parameterization of ∂S have the induced orientation from the orientation on S?

(3) Use the definition of "line integral of a vector field" to write down a Calc 1-style integral equal to $\int_{\partial S} \mathbf{F} \cdot d\mathbf{s}$.

(continued on next page)

(4) Provide a parameterization of the surface S.

(5) Use your parameterization to write down a Calc 1-style integral equal to $\iint_S \mathbf{F} \cdot d\mathbf{S}$. (Do not use Stokes' Theorem.)

Problem 3: Pick **one** of the following theorems, **give** a detailed statement of it and **explain** why it is true, with as much care as you can.

(1) If **F** is a C^1 vector field on an open region U such that **F** has path independent line integrals, then **F** is conservative.

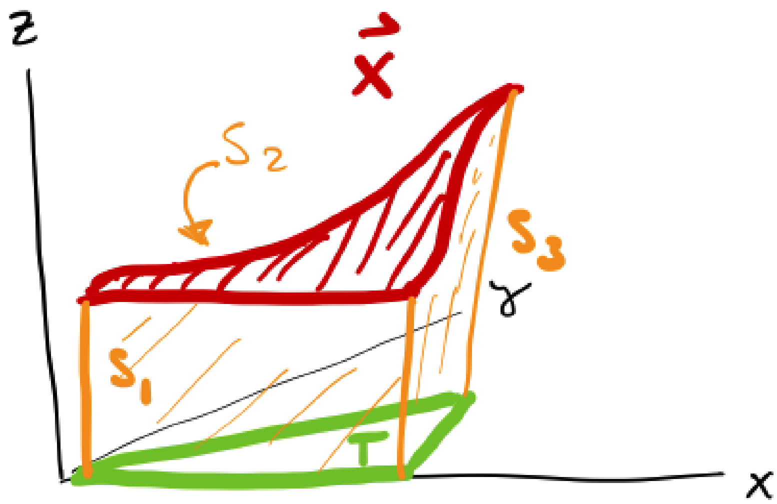
(2) Poincaré's Theorem

(3) Green's Theorem

(4) Stokes' Theorem

Problem 4: Give a parameterization of the surface of revolution obtained by rotating the image of the curve $\gamma(t) = \begin{pmatrix} \sin t \\ t+5 \\ 0 \end{pmatrix}$ for $0 \leq t \leq 4\pi$ around the y-axis.

Problem 5: Consider the surface $\mathbf{X}(s,t) = \begin{pmatrix} s \\ t \\ \sin(st)+1 \end{pmatrix}$ for points (s,t) in the triangle described by $0 \leq t \leq s$ and $0 \leq s \leq 1$. Note that this is the graph of the function $f(s,t) = \sin(s,t)$ over the indicated triangle. The surface $\mathbf{X}$ is pictured below in red; the surfaces T, S_1, S_2, and S_3 show up later in the problem.



The surface $\mathbf{X}$ has normal vector:

$$\mathbf{N}(s,t) = \begin{pmatrix} -t\cos(st) \\ -s\cos(st) \\ 1 \end{pmatrix}$$

Let $\mathbf{F}(x,y,z) = \begin{pmatrix} -y \\ x \\ 0 \end{pmatrix}$.

(questions on next page)

(1) Is the surface **X** smooth? Why or why not?

(2) Compute the circulation of **F** over **X** using any method you want.

(3) Let P be the "wavy triangular prism" which is the union of the image of the parameterized surface **X**, the triangle

$$T = \{(x, y, 0) : 0 \leq y \leq x \text{ and } 0 \leq x \leq 1\}$$

and the squares making up the sides:

$$\begin{aligned} S_1 &= \{(x,0,z) : 0 \leq x \leq 1, 0 \leq z \leq 1\} \\ S_2 &= \{(x,x,z) : 0 \leq x \leq 1, 0 \leq z \leq \sin(x^2)+1\} \\ S_3 &= \{1,y,z) : 0 \leq y \leq 1, 0 \leq z \leq \sin(y)+1\} \end{aligned}$$

Orient P with outward pointing normals.

Suppose that **G** is a $\mathbf{C}^1$ vector field such that $\operatorname{div}\mathbf{G}(x,y,z) = 10$. Write down a Calc 1 style integral equal to the flux of **G** through P. You may use any method you wish to come up with such an integral.

Problem 6: (Extra-credit) Let $\mathbf{F}(x,y) = \begin{pmatrix} M(x,y) \\ N(x,y) \end{pmatrix}$ be a C^1 vector field defined on all of $\mathbb{R}^2$. Suppose that $\mathbf{F}$ has path independent line integrals. Give a thorough explanation of how to define a potential function for $\mathbf{F}$.