MA 262	**Vector Calculus**	**Spring 2023**
F23 MA 262	**Exam 1**	**Study Guide**

The exam will be given during class. It will consist of approximately 10 questions. On average each question will take a very well-prepared student about 5 minutes (i.e. like a quiz question).

In addition to this study guide, you should study your course notes, quizzes, review sheets, and homework assignments. With regard to the homework, keep in mind that some problems are simply not suitable for an exam, while others could be adapted to be an exam problem. Additionally, some problems are extremely difficult and time-consuming the first time you encounter them, but quick and much easier the second time you encounter them. Some problems, on the other hand, are difficult and time-consuming even the second time you do them. Such problems do not make good exam questions.

You do not need to memorize the formulas for converting to polar, cylindrical, or spherical coordinates or the formula for the determinant of a 3×3 matrix. You should know how to write down a derivative matrix, the statement of the chain rule, the statement of the change of variables theorem, and how to compute arc length (among much else).

The general topics covered by the exam are listed below. For each, several open-ended questions are listed. Use these questions to make your own study guide for the more conceptual parts of the course. One or more of these open-ended conceptual questions may appear on the exam.

Conceptual Study Guide:

(1) **The derivative as a matrix**.

For a function $f\colon \mathbb{R}^n \to \mathbb{R}^m$ why is the derivative a matrix? Why does it have dimensions $m \times n$? What is the linear approximation to f based at a point? How can we use the derivative to better understand what the function does? In the case when $m = n = 2$, what does the determinant of the derivative measure and why?

(2) **The Chain Rule**

The chain rule says that the derivative of the composition of two differentiable functions is the product of the derivatives of the functions, why would we expect this to be true? If $T\colon \mathbb{R}^n \to \mathbb{R}^m$ is differentiable and if $\gamma\colon [a, b] \to \mathbb{R}^n$ is a differentiable curve, then how does the transformation affect the curve γ and how does it affect the direction and speed that γ is heading at a particular time t?

(3) **The Change of Variables Theorem**

What is the point of the theorem and how would we know that we should use it? In the statement of the theorem, the determinant of the derivative of a transformation shows up – why? We generally evaluate double (or triple) integrals by converting them to iterated Calc 1 style integrals – how do we determine the bounds for those integrals, both before and after a transformation? What is a detailed, but somewhat informal, rationale for believing the theorem?

(4) **Parameterized Curves**

Explain why the length of a curve $\gamma\colon [a,b] \to \mathbb{R}^n$ is given by the integral $\int_a^b ||\gamma'(t)||\, dt$. What does it mean to say that something is intrinsic to a curve and what are some examples? Give some examples of reparameterizing a curve. What is a flow line for a vector field? What is an equipotential line for a vector field?

(5) **Integrals**

What is the precise definition of a double integral $\iint_R f\ dA$ over a region D in terms of Riemann sums? Why is the average value of a function on a region defined in terms of an integral? How does that definition relate to the definition of average as "add up the values and divide by the number of values"? What does the integral of a scalar field over a curve measure? What does the integral of a vector field over a curve measure?

Computations: You should be able to compute the following. You do not need to actually solve any integrals, but you need to get them to a point where they would make sense to a strong Calc 1 student.

- Find a parameterization of a point rotating around a circle which is itself moving along some curve (eg. the cycloid or cardiod). The ability to find exact rotation parameters will not contribute greatly to the problem grade for such a problem.
- The product of two matrices, if it exists.
- The derivative and linear approximation of a transformation
- Determine if a C^1 function $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is orientation-preserving or orientation-reversing at a particular point. Determine how it scales area near a particular point. Is it possible for C^1 transformation to be orientation-reversing at some points and orientation reversing at others? If so, what else can you say about the transformation? (Hint: use the intermediate value theorem)
- The effect of a transformation on a parameterized curve and a direction vector
- An linear approximation to a function based at a particular point
- A double integral over a region
- A double integral over a region in polar coordinates
- A double integral over a region using the change of coordinates theorem
- The arclength of a curve
- The curvature of a curve
- The integral of a scalar field over a curve
- The integral of a vector field over a curve
- Determine if a particular curve is a flow line for a vector field or not.

Verifications: You should be able to verify:

- That certain quantitites or vectors associated to a curve are either intrinsic to the curve or intrinsic to the oriented curve.
- That the unit normal vector (namely $\dot{\mathbf{T}}/||\dot{\mathbf{T}}||$) to a curve is perpendicular to the unit tangent vector $\mathbf{T}$