MA 262 Guide to Computer Calculations and Visualizations

I. Overview

In this course, there are essentially five ways of using computers to perform calculations and visualizations. You are free to mix-and-match methods as you wish, but **you must say whenever you rely on a computer and you must specify what software you are using.** I will briefly describe each of the methods and then show you how to do the computations and visualizations you will need to do in this course.

1. **Mathematica** (or Maple or MatLab): This is expensive proprietary software that is installed on computers in the Mathematics Computer Lab on the 2nd floor of Davis. I believe you are also able to download your own copy for use while you are a student at Colby. The syntax can be somewhat non-intuitive but there is a robust set of graphical user interfaces. It can be particularly challenging to display multiple overlapping plots of different types of graphic objects (for example a vector field and a curve.) It is quite easy to use it for calculating integrals.

2. **WolframAlpha**: A free online tool that is essentially the same as Mathematica, but without the interface and help tools. I recommend using this only for computing integrals.

3. **SageMath**: This is basically the computer programming language Python adapted for math use. It is free and very powerful. You can use it online at sagecell.sagemath.org (SageCell) but there is no capacity to save your work. You can use it via Jupyter Notebooks at: sage.colby.edu (if you are off-campus you will have to use VPN). Email me to be given access to this.

   SageMath can do all the math and visualizations we need in this course and will be useful in other math and science courses as well. I encourage you to use it, *but you should work through the official introductory online tutorial first.* (Note: there is a tutorial for using SageMath in Vector Calculus, but in my opinion it requires you know vector calculus first.)

   You do not really need to know how to program in order to use it, but there is a bit of a learning curve. Most of the challenges arise from the fact that it was created for all kinds of math and it sometimes requires coaxing to do your kind of math. In particular, it defaults to symbolic calculations and it has a preference for complex numbers. For example:

```
In [1]:    1  (-1)^(1/3)

Out[1]:  (-1)^(1/3)
```

To try to get it to actually compute the cube root of (-1) we append .n() to get a numerical value:

```
In [2]:    1  (-1)^(1/3).n()

Out[2]:  0.500000000000000 + 0.866025403784439*I
```

Observe that it gave us a complex number whose cube is (-1). Chances are we were hoping to just have it compute the cube root of (-1) as (-1). To work with just real numbers we have to use a different command:

```
In [3]:    1  real_nth_root(-1,3)

Out[3]:  -1
```

If we don't like the syntax, we can create our own:

```
In [4]:    1  x = var('x') #declare that x is a mathematical variable
           2  cube_root(x) = real_nth_root(x,3) #define our own function cube_root
           3  cube_root(-1)

Out[4]:  -1
```

```
In [5]:    1  cube_root(8)

Out[5]:  2
```

```
In [6]:    1  cube_root(9)

Out[6]:  9^(1/3)
```

```
In [7]:    1  cube_root(9.0)

Out[7]:  2.08008382305190
```

4. **Desmos or Geogebra** These are two popular online graphing and computation tools. They don't do a very good job plotting vector fields and their ability to compute integrals is somewhat limited, but otherwise they are fine and you may use them.

5. **Grapher:** This is free software that comes bundled with every Mac (look under Utilities). It doesn't do computations, but it is hard to surpass its visualization capabilities, especially for vector fields.

II. Computing Integrals

Here are examples of how to compute the following definite integrals:

$$\int_1^3 t^2 \, dt \qquad \int_1^3 \int_0^y x^2 \, dx \, dy \qquad \int_1^3 \cos(t^2) \, dt$$

Mathematica:

In[1]:= $\int_1^3 t^{\wedge}2 \, dt$

Out[1]= $\dfrac{26}{3}$

Use this button.

In[2]:= $\int_1^3 \int_0^y x^{\wedge}2 \, dx \, dy$

Out[2]= $\dfrac{20}{3}$

In[3]:= $\int_1^3 \text{Cos}[t^{\wedge}2] \, dt$

Out[3]= $\sqrt{\dfrac{\pi}{2}} \left(-\text{FresnelC}\left[\sqrt{\dfrac{2}{\pi}} \right] + \text{FresnelC}\left[3\sqrt{\dfrac{2}{\pi}} \right] \right)$

This is useless

In[4]:= $\text{N}\left[\int_1^3 \text{Cos}[t^{\wedge}2] \, dt \right]$

Out[4]= -0.201661

Get a numerical
approximation.

All math commands are capitalized
and use square brackets in
Mathematica

Calculator

Basic Advanced

Sin | Cos | Tan | $e^{\blacksquare}$ | $10^{\blacksquare}$
ArcSin | ArcCos | ArcTan | Log | Log10
Define Function | Clear | Table | i | j
f | g | [■] | _ | Row+ | Col+

D | $\partial_\square \blacksquare$

x | y | t | θ | ^ | Documentation
7 | 8 | 9 | / | $\blacksquare/_\square$ | $\sqrt{\blacksquare}$ | π | e
4 | 5 | 6 | × | | | ° | i
1 | 2 | 3 | − | (■) | /. | → | ∞
0 | . | N | + | {■} | , | = | !
Tab | Enter | TraditionalForm

Input from Above | Create Input Cell
Output from Above | Create Text Cell
Command Complete | Make Template

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WolframAlpha:

integrate (t^2) from t = 1 to t =3

Definite integral   More digits   ☑ Step-by-step solution

$$\int_1^3 t^2 \, dt = \frac{26}{3} \approx 8.6667$$

integrate (integrate (x^2) from x = 0 to x = y) from y= 1 to y=3

Definite integral   More digits

$$\int_1^3 \int_0^y x^2 \, dx \, dy = \frac{20}{3} \approx 6.66667$$

integrate cos(t^2) from t = 1 to t = 3

Definite integral   More digits

$$\int_1^3 \cos(t^2) \, dt = \sqrt{\frac{\pi}{2}} \left(C\left(3\sqrt{\frac{2}{\pi}} \right) - C\left(\sqrt{\frac{2}{\pi}} \right) \right) \approx -0.201661$$

$C(x)$ is the Fresnel C integral

SageMath

```
In [10]:    1  t, x, y=var('t x y')
            2  integral(t^2, t, 1, 3)

Out[10]:  26/3

In [11]:    1  a(y) = integrate(x^2, x, 0, y)
            2  integrate(a(y),y, 1, 3)

Out[11]:  20/3

In [12]:    1  b = integrate(cos(t^2), t, 1, 3)
            2  b.n()

Out[12]:  -0.201660680170004
```

Desmos

Just type "int" to get the integral sign to appear

This button toggles between exact and approximate values

This is an approximate value, but Desmos doesn't indicate that

III. Plotting a curve on a vector field

We plot the vector field $F(x,y) = (x^2, 3x + y)$ and the curve $c(t) = (\cos(t), t\sin(t))$

Mathematica

In[7]:= `VectorPlot[{x^2, 3 x + y}, {x, -3, 3}, {y, -3, 3}]`

Out[7]=
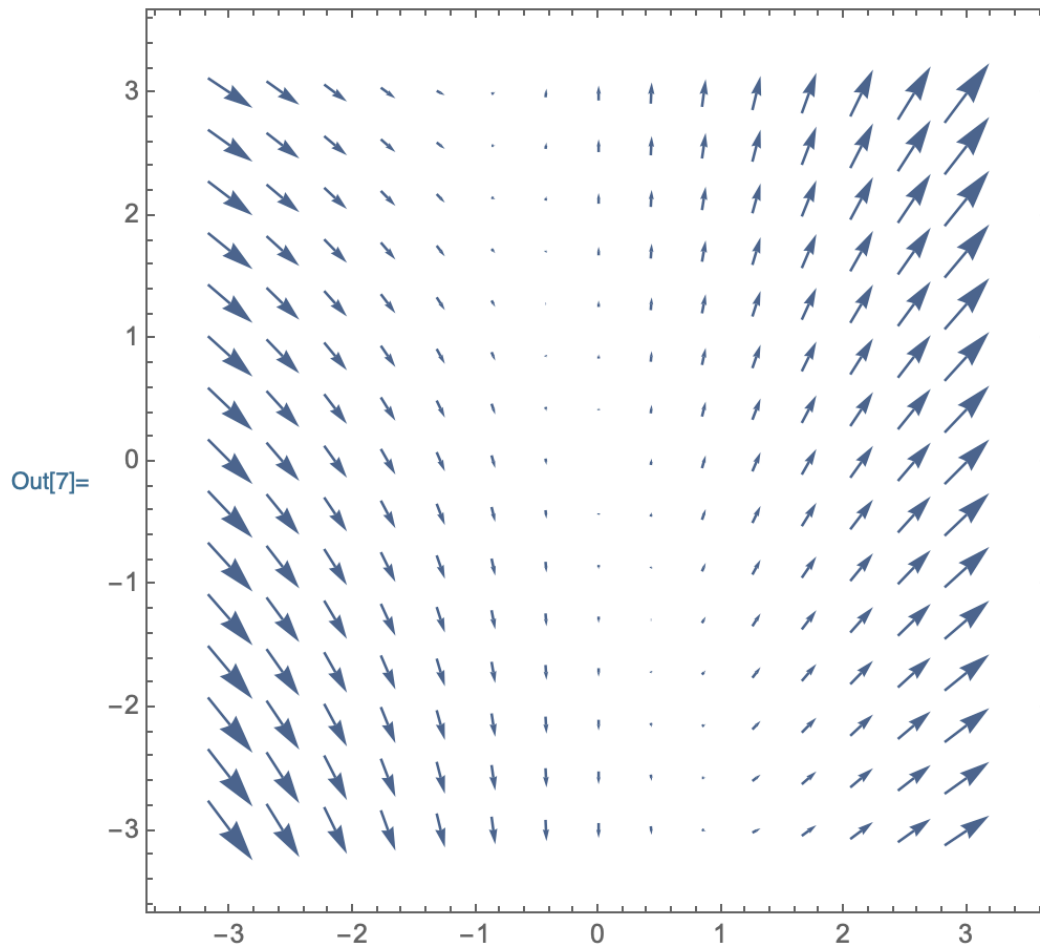
Semicolon suppresses output from this command

```
In[29]:= p = VectorPlot[{x^2, 3 x + y}, {x, -3, 3}, {y, -3, 3}];
         q = ParametricPlot[{Cos[t], t Sin[t]}, {t, 0, 2 Pi},
             PlotStyle → Red, PlotRange → {-3, 3}];
         Show[p, q]
```

Out[31]=
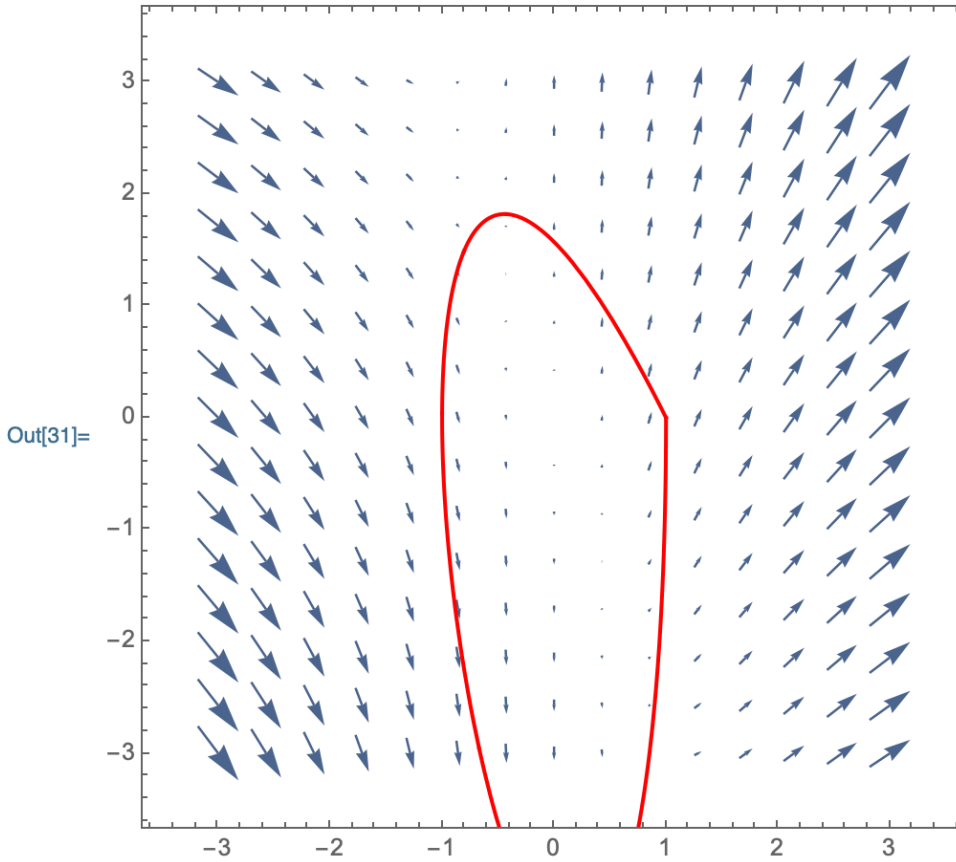
WolframAlpha

I couldn't figure out how to do this in WolframAlpha, though I could get it to plot each of the parametric plot and the vector field plot individually.
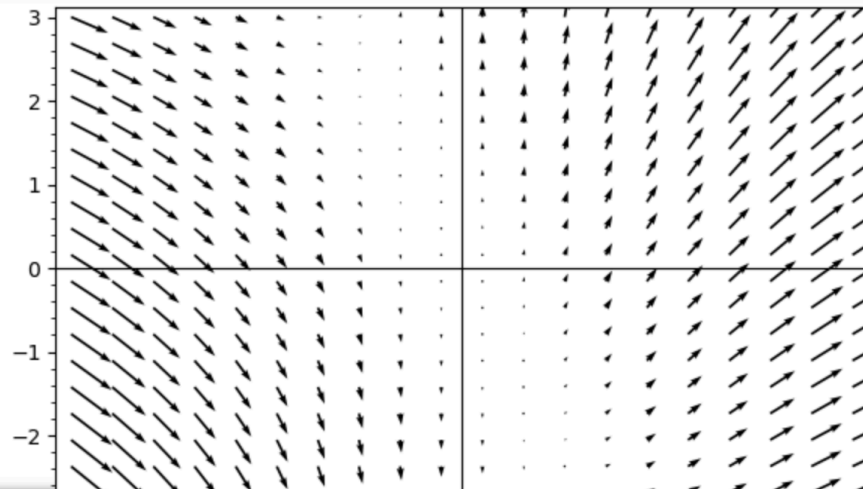
SageMath

Always declare your variables

```
In [1]:   1  x, y, t = var('x y t')
```

```
In [4]:   1  F(x,y) = (x^2, 3*x + y)
          2  plot_vector_field(F, (x, -3, 3), (y, -3,3))
```
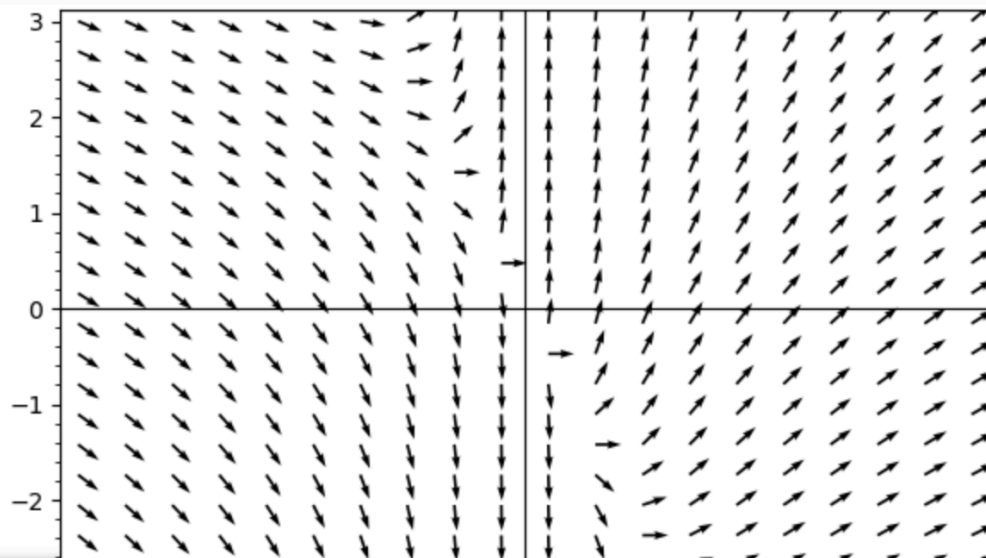
Out[4]:

It looks better if we plot all arrows the same length

```
In [3]:   1  plot_vector_field(F/F.norm(), (x, -3, 3), (y, -3,3))
```
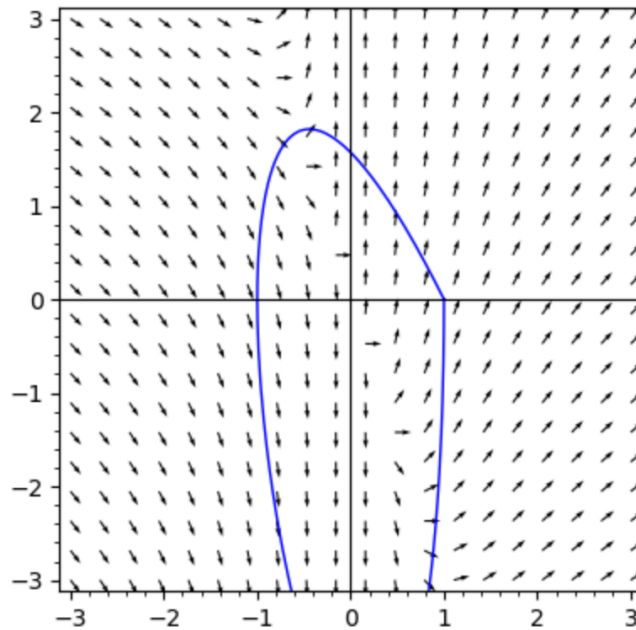
Out[3]:

Here is a parametric plot on top of a vector field plot.
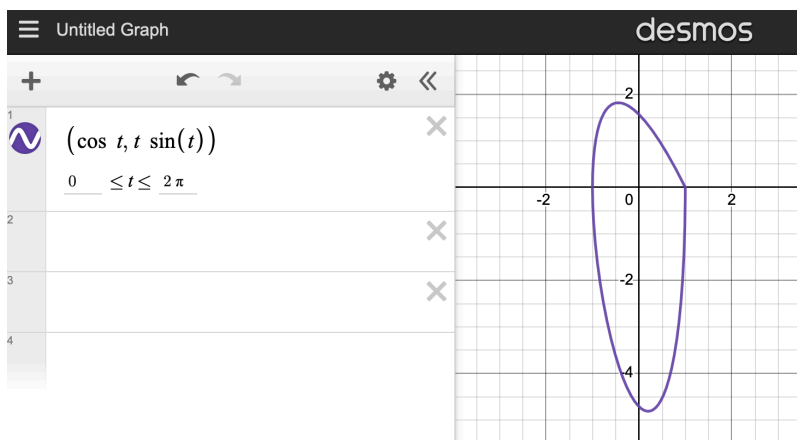
```
In [5]:    1  a = plot_vector_field(F/F.norm(), (x, -3, 3), (y, -3,3))
           2  b = parametric_plot(c(t), (t,0,2*pi), xmin=-3, xmax=3, ymin=-3, ymax=3)
           3  show(a + b)
```


Desmos & Geogebra

There does not seem to be a great way to visualize vector fields in Desmos or Geogebra, thought some people have written scripts to do so.
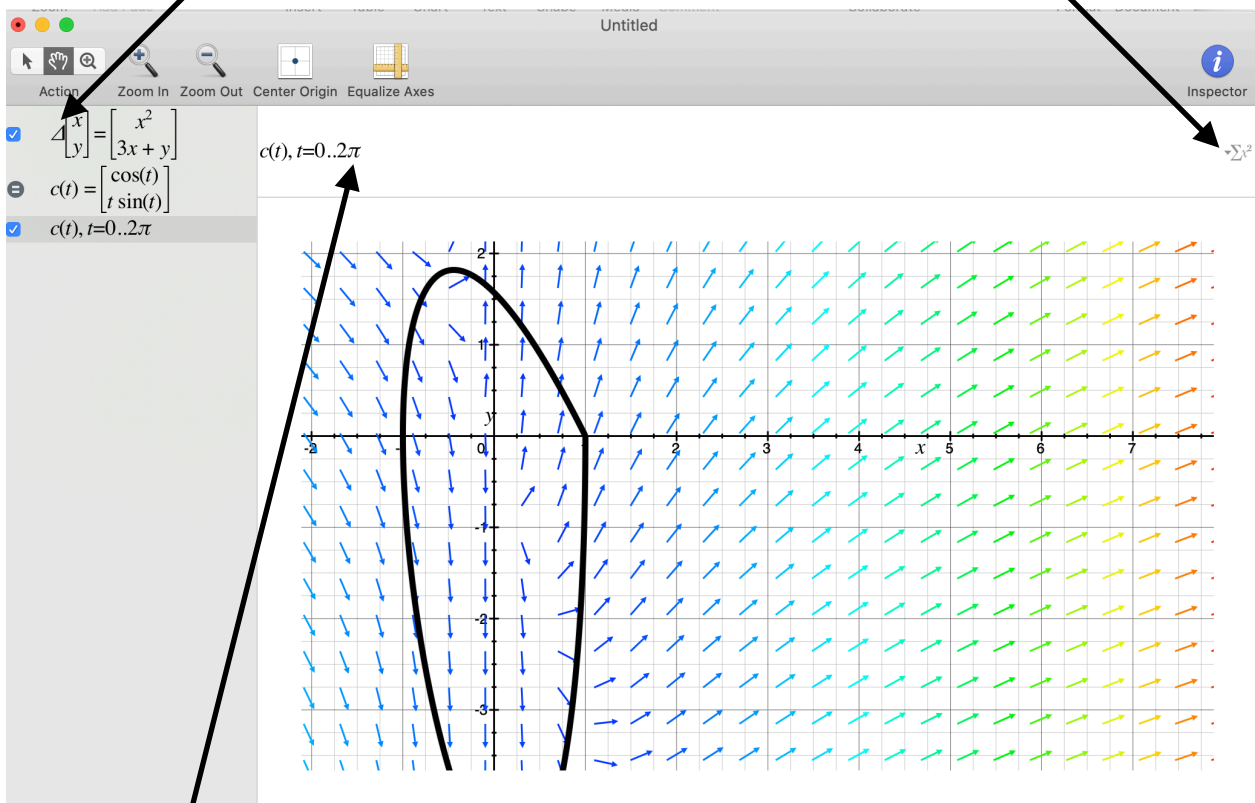
Here is an example of a parametric plot in Desmos:

Grapher

Here is an example of how to plot vector fields and parametric curves in Grapher:

To get the Greek letter, just type its name: "Delta"

Use this menu to get the brackets for vectors

Type in this window. Initially it will say "y=" but you can delete that.

ChatGPT:

Don't even consider using it — it thinks it knows math, but it doesn't:

Scot integrate t^2 from t=1 to t = 3

The definite integral of t^2 from t=1 to t=3 can be found using the power rule of integration, which states that the derivative of t^n is nt^(n-1). The result is:

∫[1,3] t^2 dt = (t^3/3) | [1,3] = (3^3/3) - (1^3/3) = 8.

So the definite integral of t^2 from t=1 to t=3 is equal to 8.

Scot integrate cos(t^2) from t =0 to t = 3

An error occurred. If this issue persists please contact us through our help center at help.openai.com.

There was an error generating a response