MA 314 Homework 6: When a straight line isn't the shortest distance...

This homework is intended to be lighter than usual. The goal is to get you used to working with surfaces.

1. Reading

Read the following sections on hyperbolic geometry:

- Bonahon: Sections 2.1 - 2.2
- Schwartz: Section 10.3

2. Mortar

In your reading, the hyperbolic plane is defined as the set

$$\mathbb{H}^2 = \{(x,y) \in \mathbb{R}^2 : y > 0\}$$

with arc length of a parameterized C^1 curve $\gamma(t) = (x(t), y(t))$ for $a \leq t \leq b$ defined by

$$l_{\text{hyp}} = \int_a^b \frac{||\gamma'(t)||}{y(t)}\, dt.$$

(1) Summarize the proof of the result (Lemma 2.1) that l_{hyp} is a path metric.

(2) Sketch a figure in the hyperbolic plane and its image under the standard inversion (Section 2.2.3 of Bonahon)

(3) (Homotheties) Let $\lambda > 0$ be fixed. Let $f(x,y) = (\lambda x, \lambda y)$. Let $\gamma\colon [a,b] \to \mathbb{H}^2$ be a C^1 curve. Prove that γ and $f \circ \gamma$ have the same hyperbolic length.

(4) (Horizontal translations) Let x_0 be fixed. Let $g(x,y) = (x + x_0, y)$. Let $\gamma\colon [a,b] \to \mathbb{H}^2$ be a C^1 curve. Prove that γ and $g \circ \gamma$ have the same hyperbolic length.

(5) Let $Q = [0,1] \times [1,2]$ and $Q' = [0,1] \times [2,3]$ be squares in $\mathbb{H}^2$. Compute their hyperbolic area (for the formula for hyperbolic area, see the end of the Section 10.3 of Schwartz. You'll have to remember how to compute a double integral. Probably you'll need to review Fubini's theorem to remind yourself how to do that.) Comment on the result of your calculations.