MA 314 Homework 5: Round and Round

1. Reading

We now begin our study of spherical geometry. Read:

- Schwartz: 9.1 - 9.2
- Bonahon: Chapter 3
- Steven Strogatz's blog post on spherical geometry "Think Globally":
  `http://opinionator.blogs.nytimes.com/2010/03/21/think-globally/?_php=true&_type=blogs&_r=0`
  Complete the blog reading response.

2. Reading Response

(The purpose of this blog reading is twofold: First, you get to read an example of a well-written blog essay on a geometric topic – should be helpful for when you have to write your own! Second, you get to learn some interesting facts about geodesics, a central concept of this course. The questions below are intended to have you explicitly consider the qualities of good mathematical writing.)

(1) Share some of the interesting things you learned from the essay.

(2) What questions does the essay provoke? What would you be interested in pursuing further?

(3) Who is the intended audience of the essay? What qualities of the writing make the author successful or unsuccessful at reaching that audience.

(4) Choose two other possible audiences for a blog post on the same topic as the essay you read. Say what those audiences are and, for each, how the essay might be structured differently in tone and content if it were written for that audience.

3. Mortar

(1) Explain why the sphere S^2 (with the "round" metric) is homogenous and isotropic.

(2) Give an explicit parameterization for a geodesic on S^2.

4. BRICKS

(1) Consider the space (the torus) obtained by starting with the square $[-1,1] \times [-1,1]$ in $\mathbb{R}^2$ and working up the equivalence relation generated by:

$$\begin{array}{rcl} (\pm 1, y) & \sim & (\mp 1, y) \\ (x, \pm 1) & \sim & (x, \mp 1) \\ (x, y) & \sim & (x, y) \end{array}$$

(i.e. $\sim$ is obtained from the above formulas by also insisting on reflexivity, symmetry, and transitivity) This space is described in Figure 1.1 on page 2 of Schwartz's book.

It can also be obtained by instituing the equivalence relation on $\mathbb{R}^2$ where to points (x,y) and (a,b) are equivalent if and only if their first coordinates differ by an even integer and their second coordinates differ by an even integer.

Your answers to the next questions will necessarily be informal since we haven't yet given a precise definition of the metric on the space. Nonetheless you should be able to find the keys to following questions:

(a) Describe the isometries of this space which are equivalent to horizontal and vertical translations.

(b) Find some isometries of this space which are not the equivalent of horizontal and vertical translations. (Hint: think about symmetries of the square.)

(c) Explain why this space is homogeneous but not isotropic.