MA 314 Homework 15: Bad Quotient! Bad! Bad!

1. Reading

(1) Read some sources which you'll use as the basis for your blog essay.

2. Bricks

(1) In class we considered two bad quotients:

- $X = \mathbb{R}$ with the equivalence relation: $x \sim y \Leftrightarrow (x - y) \in \mathbb{Q}$. This has the property that in $\overline{X}$ there are distinct points $\overline{P}$ and $\overline{Q}$ such that the chain $\overline{C} : \overline{P}, \overline{Q}$ of jump number 1 has length 0.
- $X = [0,1] \times [-1,1] \subset \mathbb{R}^2$ with the equivalence relation generated by $(1/n, 1) \sim (1/n, -1)$. This has the property that in $\overline{X}$ there are distinct points $\overline{P}$ and $\overline{Q}$ such that for any $\varepsilon > 0$, there is a chain $\overline{C} : \overline{P}, \overline{x}, \overline{Q}$ of jump number 2 with $\ell(\overline{C}) < \varepsilon$. But there there exists an $\varepsilon > 0$ such that no chain of jump number less than 2 (i.e. jump number 1) has length less than ε.

Construct a space X and an equivalence relation giving rise to a quotient space $\overline{X}$ such that both of the following hold:

(a) There exist distinct points $\overline{P}, \overline{Q} \in \overline{X}$ such that for any $\varepsilon > 0$, there is a chain $\overline{C}$ from $\overline{P}$ to $\overline{Q}$ of jump number $j = 3$ such that $\ell(\overline{C}) < \varepsilon$.

(b) For all pairs of distinct points $\overline{P}, \overline{Q} \in \overline{X}$ there exists $\varepsilon > 0$ such that if $\overline{C}$ is a chain from $\overline{P}$ to $\overline{Q}$ of jump number $j(\overline{C}) < j$ then $\ell(\overline{C}) > \varepsilon$.