MA 314 Homework 14: Surfacing for air

1. Reading

(1) Read Bonahon Sections 4.1 and 4.2 and Schwartz Sections 3.1, 3.2, and 3.3.

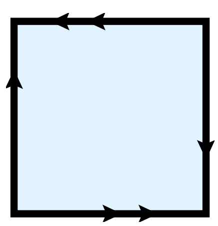
(2) Suggest some sources which you'll use as the basis for your blog essay.

2. Mortar

(1) Bonahon defines the length of a discrete walk and then defines distance in the the quotient space as the infimum of lengths of discret walks. Schwartz defines chains and pathifications. Articulate how these definitions do or don't do the same thing. Are they equivalent concepts?

3. Bricks

(1) Prove that the result of removing an open disc from the projective plane is a Möbius band. If it helps, the projective plane is pictured below:



(2) Let X be the strip in $\mathbb{R}^2$ bounded by the lines $x = -1$ and $x = +1$. For a fixed $r \geq 0$, let $\overline{X}_r$ be the result gluing the point $(-1, y)$ to $(1, y+r)$ for each $y \in \mathbb{R}$. We can give $\overline{X}_r$ a path metric inherited from X as discussed in class for the torus. For the following problems, you may informally. Recall that a geodesic is a locally length-minimizing path. The geodesics in $\overline{X}_r$ are all unions of straight line segments in X.

(a) For $r = 0$, sketch a picture of a geodesic loop in $\overline{X}_0$. What is its length? Are there any other geodesic loops?

(b) For $r = 1$, sketch a picture of a geodesic loop in $\overline{X}_1$. What is its length? Are there any other geodesesic loops?

(c) Prove that if $r, s \geq 0$ with $r \neq s$, then $\overline{X}_r$ is not isometric to $\overline{X}_s$.

(d) Describe the isometries of $\overline{X}_r$ for different values of r. Do the $\overline{X}_r$ for different values of r all have similar types of isometries?