The internet and the hyperbolic plane

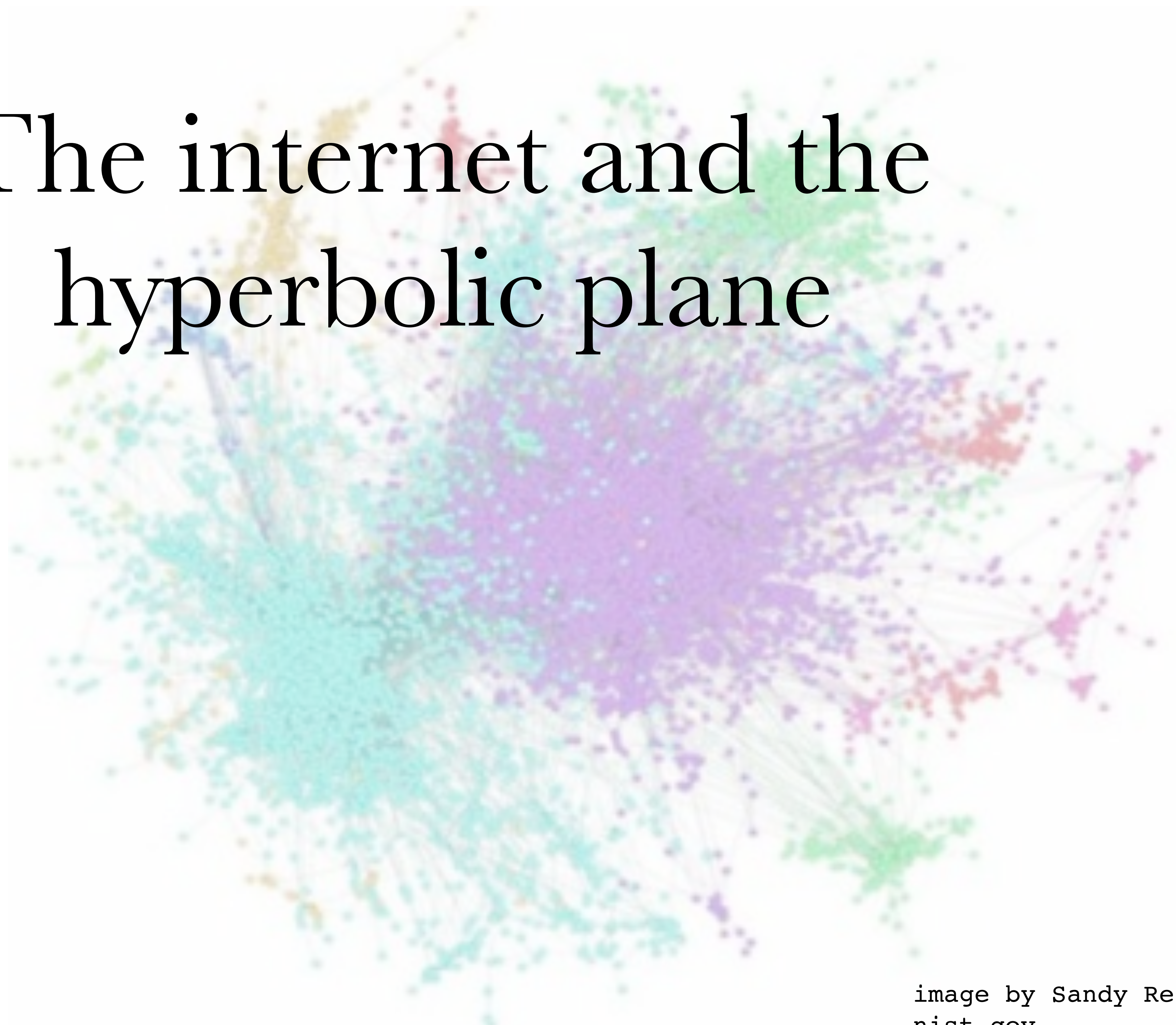
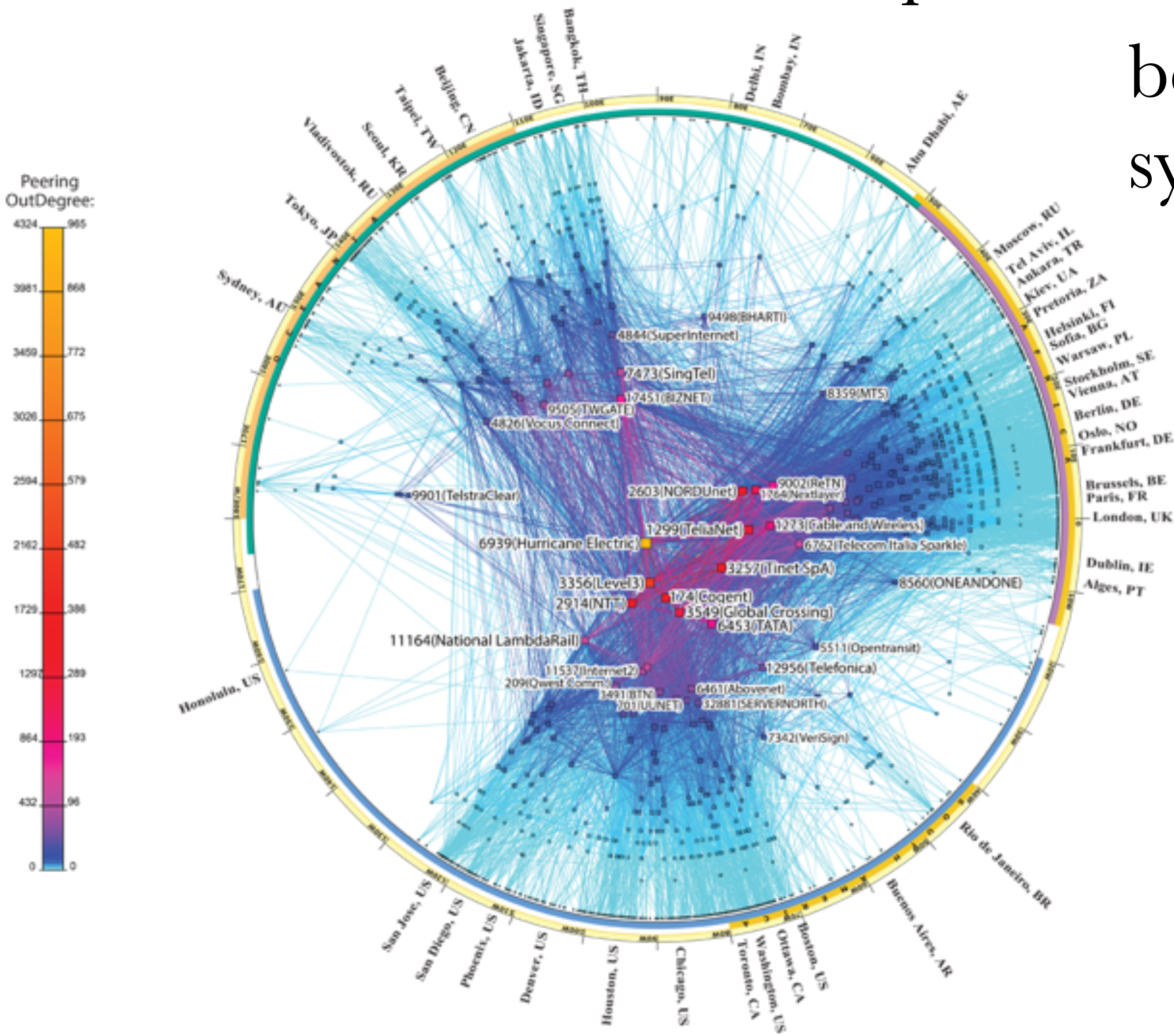


image by Sandy Ressler
nist.gov

The internet infrastructure is composed of connections between autonomous systems (AS).



http://www.caida.org

The number of ASs increases by approx 2400/year.

To route information between locations, the ASs must discover the best path to each possible destination.

The constantly changing structure of the internet leads to immense and quickly growing routing costs: the existing internet routing architecture may not last the decade.

Roads without an atlas

- 22 State Route 22
- 23 State Route 23
- 24 State Route 24
- 25 State Route 25
  - 25 SR 25 Business
- 26 State Route 26
- 26A State Route 26A
- 27 State Route 27
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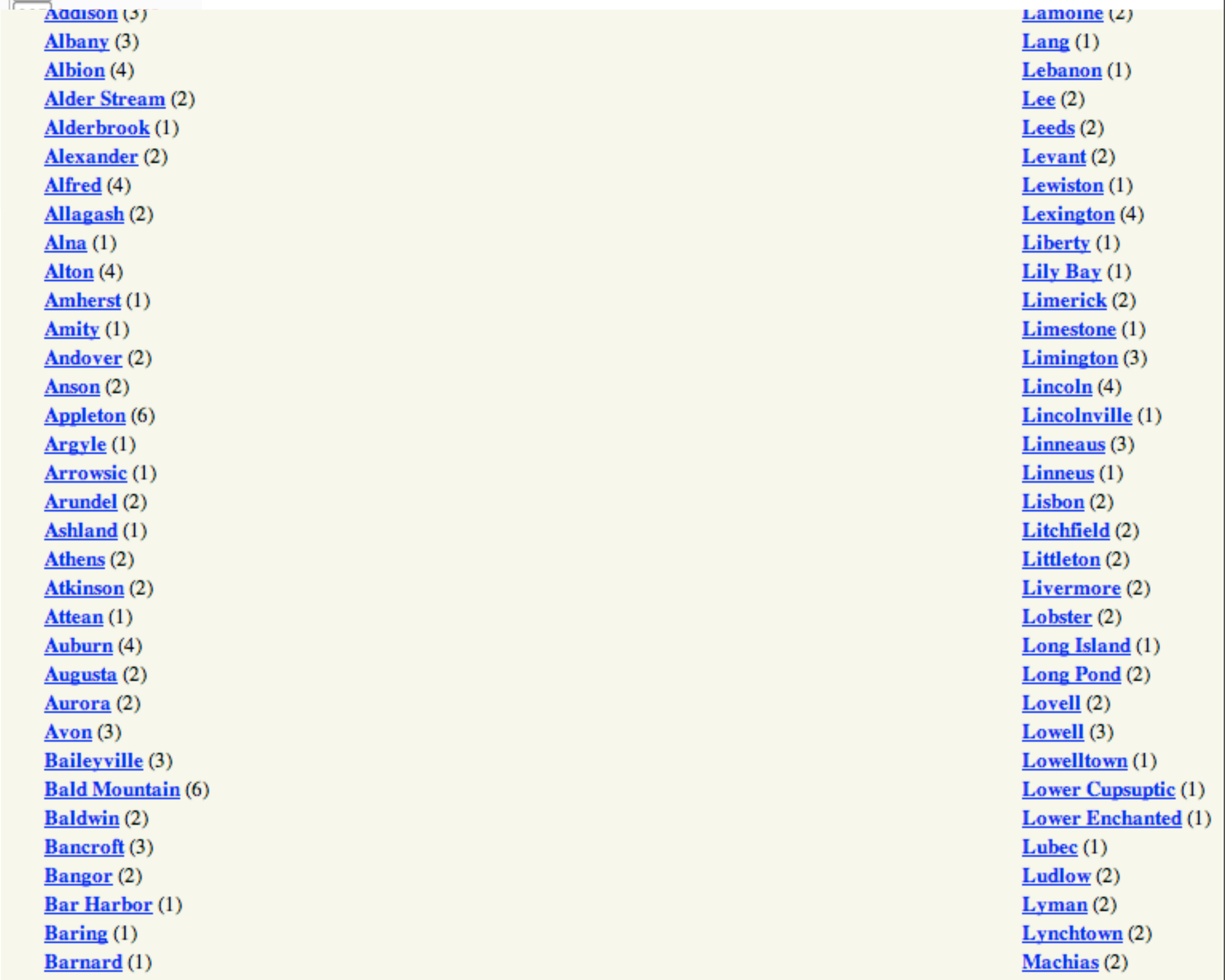
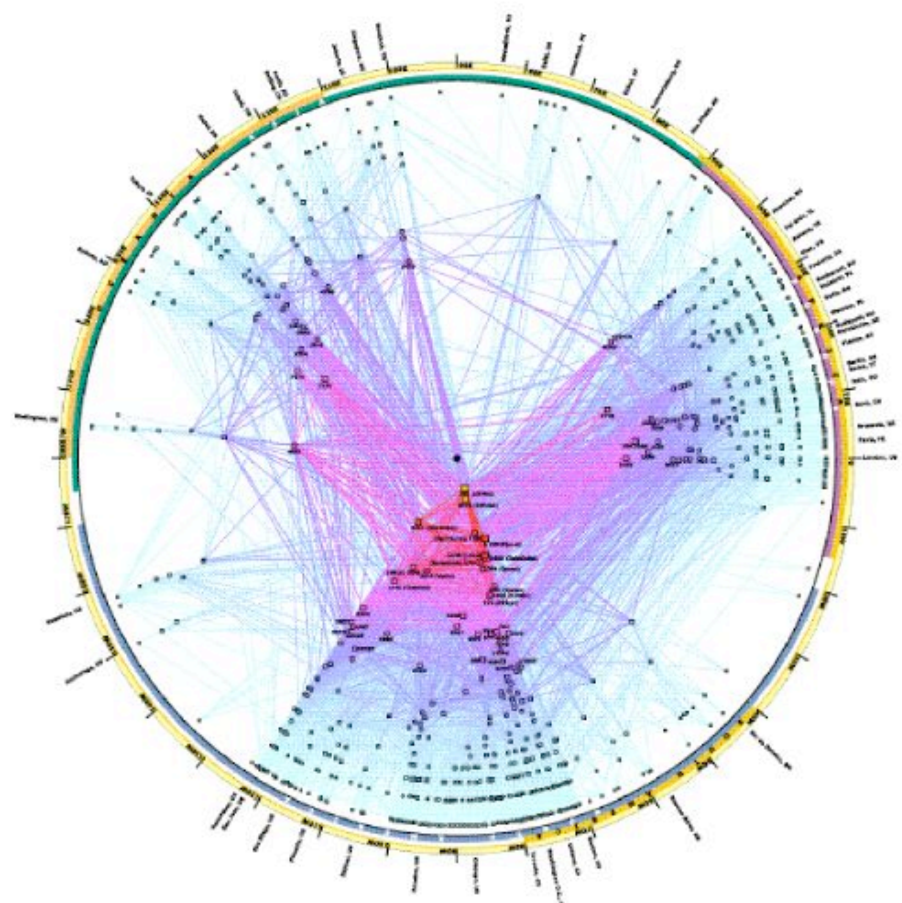


Addison (3)
Albany (3)
Albion (4)
Alder Stream (2)
Alderbrook (1)
Alexander (2)
Alfred (4)
Allagash (2)
Alna (1)
Alton (4)
Amherst (1)
Amity (1)
Andover (2)
Anson (2)
Appleton (6)
Argyle (1)
Arrowsic (1)
Arundel (2)
Ashland (1)
Athens (2)
Atkinson (2)
Attean (1)
Auburn (4)
Augusta (2)
Aurora (2)
Avon (3)
Baileyville (3)
Bald Mountain (6)
Baldwin (2)
Bancroft (3)
Bangor (2)
Bar Harbor (1)
Baring (1)
Barnard (1)

Lamoine (2)
Lang (1)
Lebanon (1)
Lee (2)
Leeds (2)
Levant (2)
Lewiston (1)
Lexington (4)
Liberty (1)
Lily Bay (1)
Limerick (2)
Limestone (1)
Limington (3)
Lincoln (4)
Lincolnville (1)
Linneaus (3)
Linneus (1)
Lisbon (2)
Litchfield (2)
Littleton (2)
Livermore (2)
Lobster (2)
Long Island (1)
Long Pond (2)
Lovell (2)
Lowell (3)
Lowelltown (1)
Lower Cupsuptic (1)
Lower Enchanted (1)
Lubec (1)
Ludlow (2)
Lyman (2)
Lynchtown (2)
Machias (2)

We need a map!

Goal: find a path metric space X so that the internet graph can be drawn in X and so that the best path in the graph is approximately a geodesic in X.

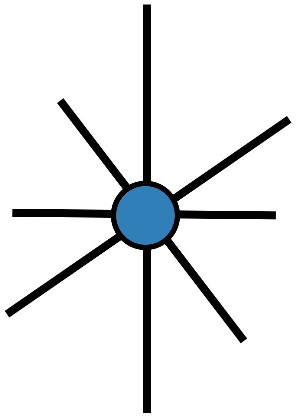


What are the essential properties of the AS graph?

The graph is <u>scale-free</u>

$$P(k) = \frac{\#\text{ nodes w/ degree } k}{\#\text{ nodes}}$$

$$\approx \frac{1}{k^{\gamma}} \quad \text{with} \quad \gamma \geq 2$$



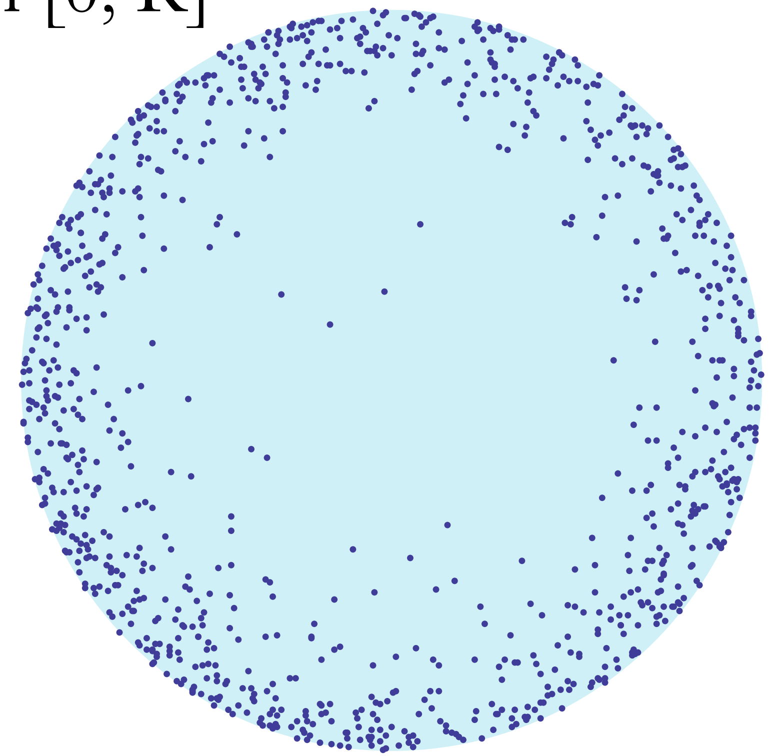
degree = 8

This strongly corresponds to robustness to failure.

The hyperbolic plane can be used to create scale-free networks.

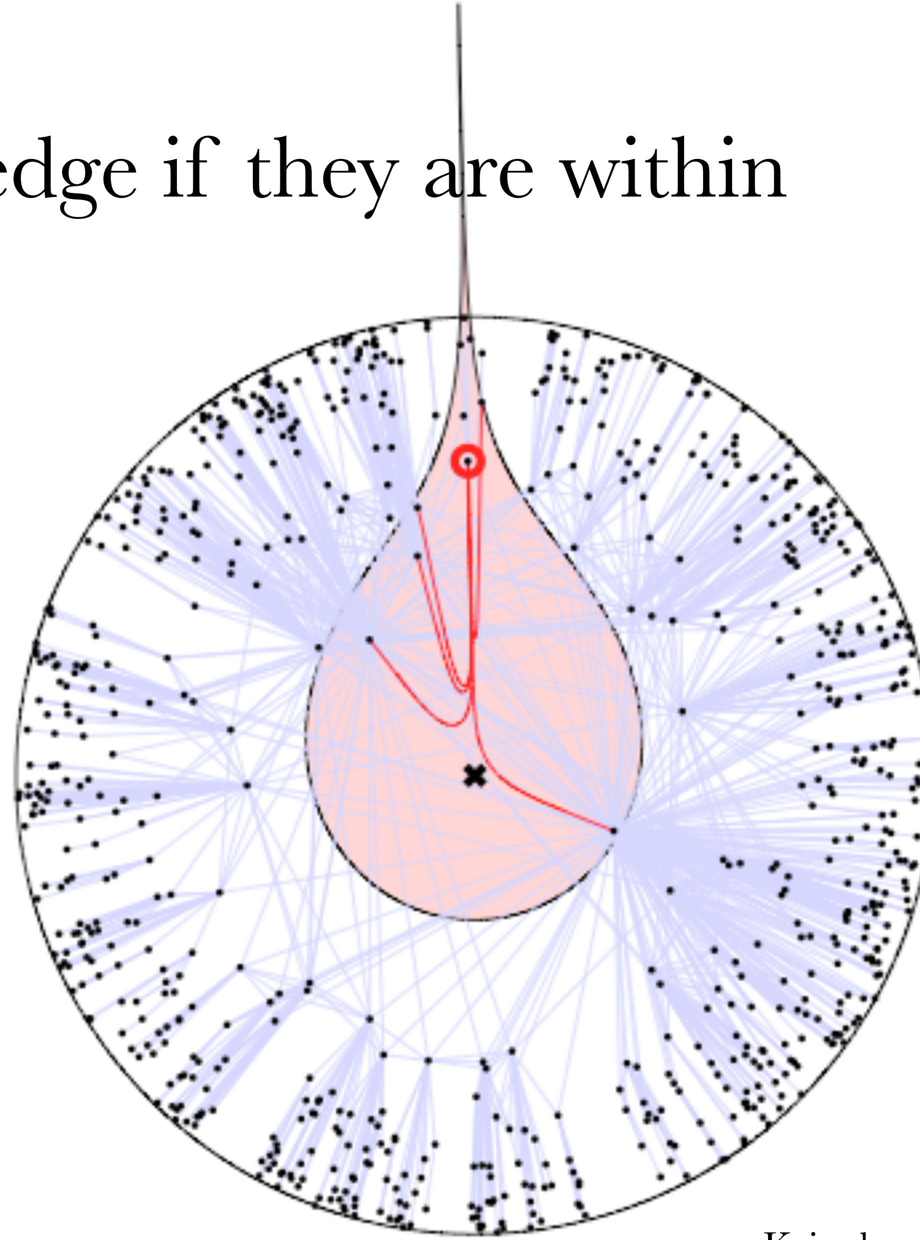
Randomly distribute N nodes in a hyperbolic disc of radius R.

- Angles are uniformly distributed in $[0, 2\pi]$.
- Hyperbolic radii are distributed in [0, R] with PDF $\dfrac{\sinh(r)}{\cosh(R) - 1}$



The hyperbolic plane can be used to create scale-free networks.

- Join two vertices by an edge if they are within hyperbolic distance R



Krioukov, et al.

Area of intersection decreases exponentially with *r*.

$$\bar{k}(r) \approx e^{-r/2}$$

Inverting we find:

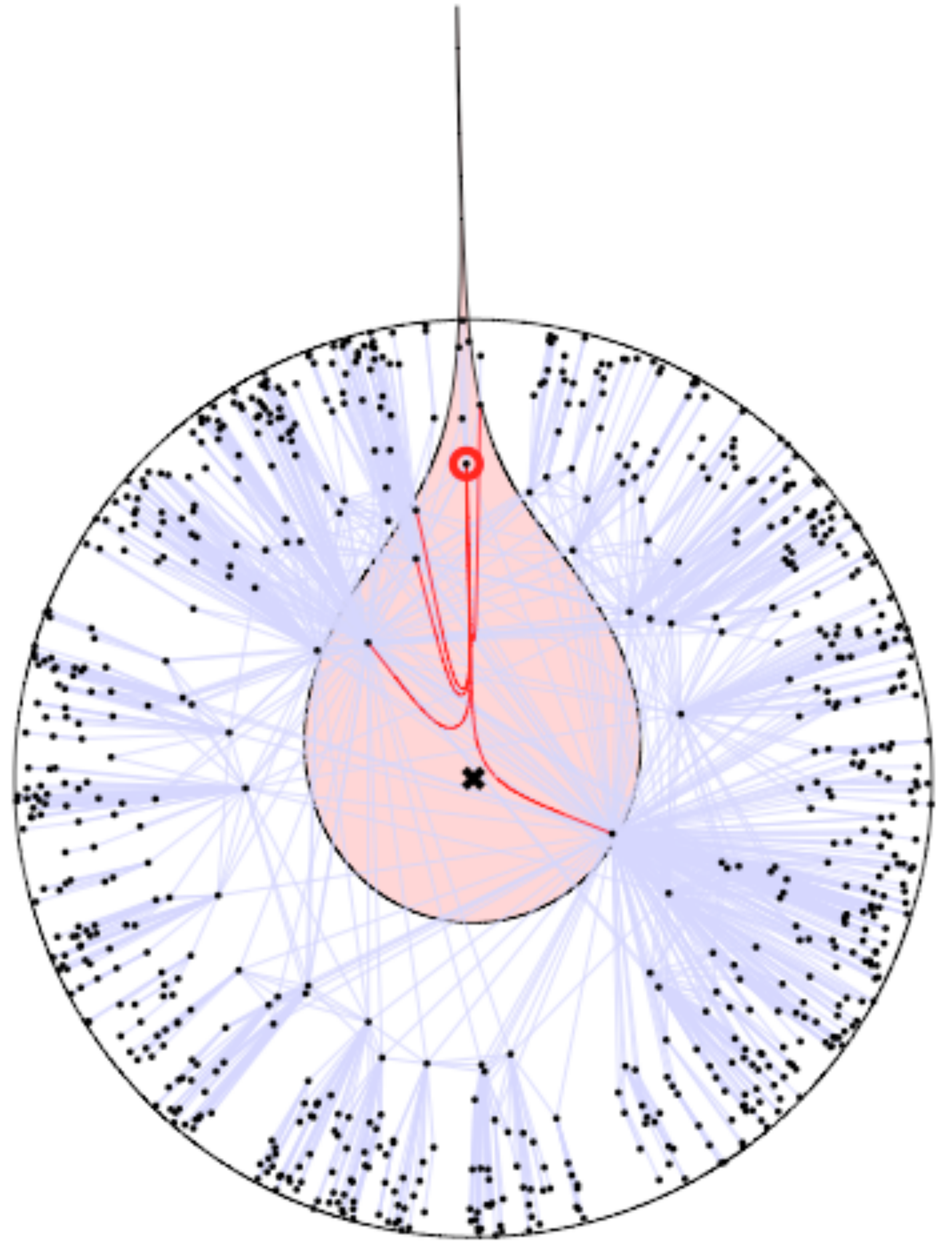
$$\bar{r}(k) \approx -2\ln(k)$$

Recalling:

$$\rho(r) \approx e^{r-R}$$

We have:

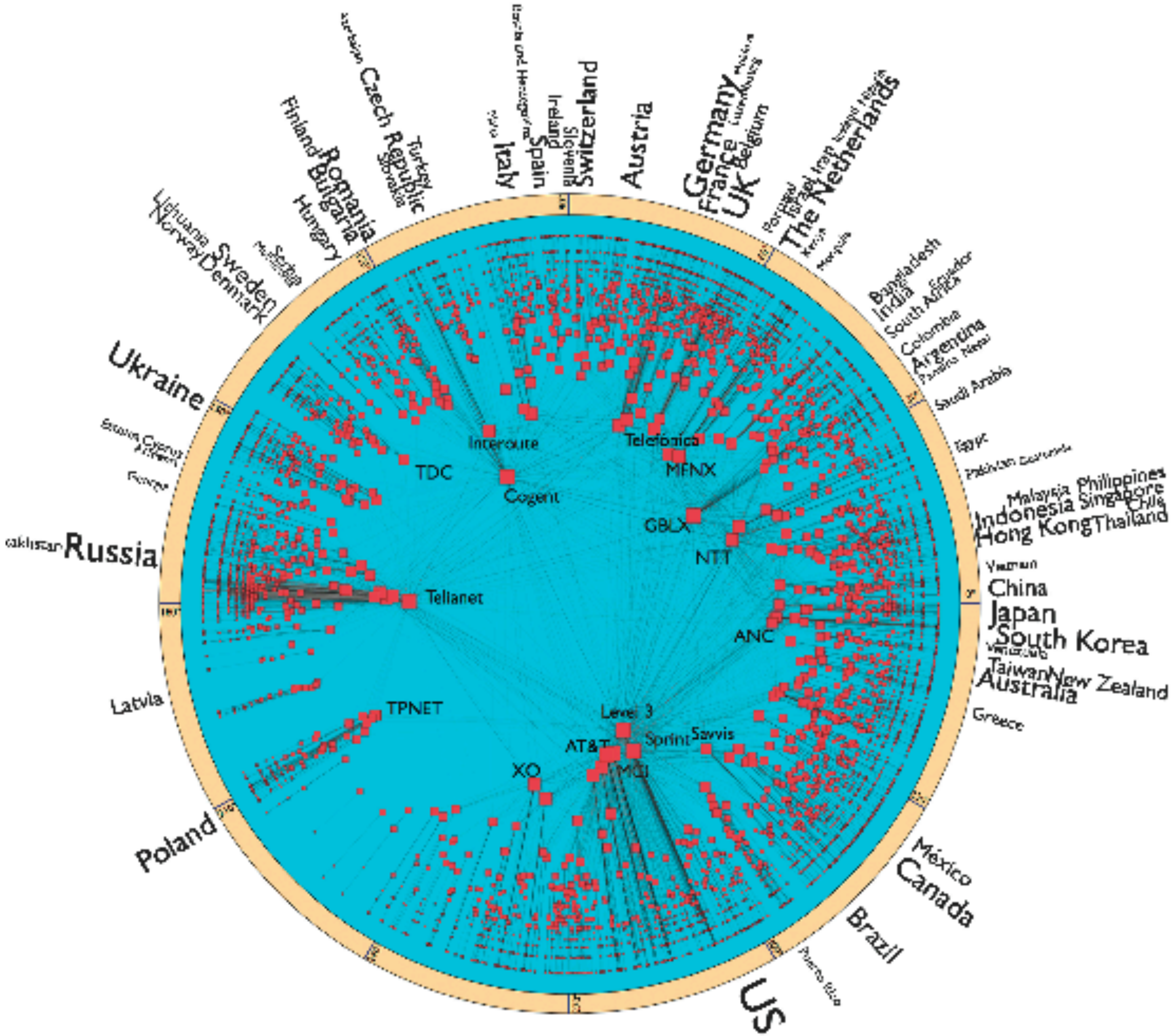
$$\rho(\bar{r}(k))|\bar{r}'(k)| \approx \frac{2}{k^3}$$



Goal: find a path metric space X so that the internet graph can be drawn in X and so that the best path in the graph is approximately a geodesic in X.

In fact, it is possible to place the nodes of *any* scale-free network into the hyperbolic plane so that shortest paths track geodesics.

Doing this for the AS network



Boguña, et al

Doing this for the AS network

- Produces a routing algorithm which is successful 97% of the time.
- Reflects geo-political boundaries
- Does not cause abnormal traffic congestion
- Is unaffected by random removals from the graph.
- Has limited performance degradation over time.

Sources:

Krioukov, et al. *Hyperbolic Geometry of Complex Networks*
arXiv: 1006.5169

Boguña, et al. *Sustaining the Internet with hyperbolic mapping*
Nature Communications September 7, 2010.