MA 398 Homework 6: When a straight line isn't the shortest distance...

This homework is intended to be lighter than usual. The goal is to get you used to working with surfaces.

1. Reading

Read the first half or so of Milnor's article on the history of hyperbolic geometry. Along with your Huts problems, please tell me two interesting things you learned.

2. Huts

These problems are intended to give you some practice with basic concepts. They will often involve calculation, rarely involve new ideas, and won't be graded. However, your answers will be collected!

In class we said that the hyperbolic plane was the set

$$\mathbb{H}^2 = \{(x,y) \in \mathbb{R}^2 : y > 0\}$$

with arc length of a parameterized C^1 curve $\gamma(t) = (x(t), y(t))$ for $a \leq t \leq b$ defined by

$$l_{\text{hyp}} = \int_a^b \frac{||\gamma'(t)||}{y(t)}\, dt.$$

These two problems ask you to investigate two isometries of $\mathbb{H}^2$. If you need a hint, read section 2.2.1 of Bonahon.

(1) (Homotheties) Let $\lambda > 0$ be fixed. Let $f(x,y) = (\lambda x, \lambda y)$. Let $\gamma\colon [a,b] \to \mathbb{H}^2$ be a C^1 curve. Prove that γ and $f \circ \gamma$ have the same hyperbolic length.

(2) (Horizontal translations) Let x_0 be fixed. Let $g(x,y) = (x + x_0, y)$. Let $\gamma\colon [a,b] \to \mathbb{H}^2$ be a C^1 curve. Prove that γ and $g \circ \gamma$ have the same hyperbolic length.