MA 302: Yet More Practice Problems **Name:**________________________

(1) Let $S \subset \mathbb{R}^3$ be an ellipsoid enclosing the origin, oriented outward. Let $P \subset \mathbb{R}^3$ be a cube enclosing the origin and enclosed by S. Orient P outward. Let $\mathbf{F}$ be an incompressible vector field defined on $\mathbb{R}^3 - \{\mathbf{0}\}$. Prove that the flux of $\mathbf{F}$ through P is the same as the flux of $\mathbf{F}$ through S.

(2) Let $\mathbf{F}\colon \mathbb{R}^3 \to \mathbb{R}^3$. Let $\mathbf{a}$ be a point in $\mathbb{R}^3$. For each $n \in \mathbb{N}$, let V_n be a compact 3-dimensional region containing $\mathbf{a}$, such that the regions V_n limit to $\mathbf{a}$. Orient the boundary of V_n outwards. Use the divergence theorem to prove that

$$\operatorname{div} \mathbf{F}(\mathbf{a}) = \lim_{n \to \infty} \frac{1}{\operatorname{vol} V_n} \iint_{\partial V_n} \mathbf{F} \cdot d\mathbf{S}.$$

(3) Let S be the box with corners $(\pm 1, \pm 1, \pm 1)$, oriented outward. Let $\mathbf{F}(x,y,z) = \begin{pmatrix} xyz \\ xy \\ z \end{pmatrix}$. Find the flux of $\mathbf{F}$ through S.

(4) Prove that inside a hollow planet there is no gravity. (You may use Gauss' Law of Gravitation.)

(5) Give a complete, precise statement of the divergence theorem.