MA 122: Weekly HW 9

Answer these questions on a separate sheet of paper. Remember that your work must be very neat and complete.

Problem A:

(1) Use differentials to estimate the error in calculating the surface area of a closed cylindrical can that has been measured to be h centimeters tall and d centimeters in diameter if the measurement error is at most .05 cm. The formula for surface area of such a can is $A = \pi hd + 2\pi(d/2)^2$. Do this for the following values of h and d:
   (a) $h = 15$, $d = 30$
   (b) $h = 27$, $d = 42$

(2) If R is the total resistance of three resistors connected in parallel with resistances R_1, R_2, and R_3, then

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_2}.$$

If the resistances are measured in ohms as $R_1 = 25\Omega$, $R_2 = 40\Omega$, and $R_3 = 50\Omega$, with a possible error of 0.5% in each case, estimate the maximum error in the calculated value of R.

Problem B:

(1) Let $\phi(t) = (\cos t + 5, 3\sin t + 3)$ and let $f(x,y) = x - y$.
   (a) Draw the image of ϕ in $\mathbb{R}^2$. That is, mark all points $(a,b) \in \mathbb{R}^2$ such that there exists $t \in \mathbb{R}$ with $\phi(t) = (a,b)$.
   (b) Find an equation for $f \circ \phi(t)$ and take its derivative (with respect to t).
   (c) Use the chain rule to find $\frac{d}{dt} f \circ \phi(t)$.

(2) Let $\phi(t) = (\cos(t), 3\sin(t))$ and let $s(x,y)$ denote the square of the distance from a point $(x,y) \in \mathbb{R}^2$ to the line $-x + 3y = 0$.
   (a) Find an equation for $s(x,y)$. (Hint: use the dot product as we did in class.)
   (b) Use the chain rule to find $\frac{d}{dt} s \circ \phi(t)$.
   (c) Find a number $t_0 \in [-\pi, \pi),$, such that $\phi(t_0)$ is the point on the curve farthest from the line defined by the equation $-x + 3y = 0$.

(3) Let $\phi(t) = (t, \cos(t)+4, \sin(t)+4)$ and let $s(x,y,z)$ denote the square of the distance from the point (x,y,z) to the plane P defined by the equation $y+z=0$. You may want to use Grapher or Mathematica[1] to draw the image of $\phi(t)$ in $\mathbb{R}^3$. Find the number t_0 such that the point $\phi(t_0)$ is the point on the image of ϕ closest to the plane P.

Problem C: A function $f(x,y)$ is **homogeneous of degree** p if for all $t,x,y \in \mathbb{R}$:

$$f(tx,ty) = t^p f(x,y).$$

Let $\mathbf{x_0} = (x_0, y_0) \in \mathbb{R}^2$. Show that any differentiable, homogeneous function of degree p satisfies:

$$\nabla f(\mathbf{x_0}) \cdot \mathbf{x_0} = p f(\mathbf{x}_0).$$

(Hint: Define $\phi(t) = t\mathbf{x}_0$ and compute $\frac{d}{dt}|_{t=1} f \circ \phi(t)$ using the chain rule.)

Problem D: Let $f(x,y) = x^4 + y^4 - 4xy + 1$.

(1) Find $f_x(x,y)$ and $f_y(x,y)$.
(2) Find all second partial derivatives of f.
(3) Find the 1st order Taylor approximation to $f(x,y)$ at $(0,0)$.
(4) Find the 2nd order Taylor approximation to $f(x,y)$ at $(0,0)$.
(5) Find all critical points of f.
(6) Classify the critical points of f using the 2nd derivative test.

Problem E: The cost of building a box depends on the h, l, and w (all measured in feet). For a particular design, materials cost 3 dollars per square foot for the base and top and 5 dollars per square foot for the sides. If the box must have a volume of 100 cubic feet, what is the cheapest it can be built for? (Hint: use the constraint that the volume is 100 ft^3 to define a function $A(h,l)$ which you will minimize.)

[1] In Grapher use "Curves" under "Create New Equation From Template". In Mathematica help look at "spacecurve".