Homework 2

You should answer these carefully and completely on a separate piece of paper. Notice that some questions have subquestions.

Problem 1: The tangent line to the graph of $f(x) = \tan(x)$ at $x = \pi/3$ has slope $m = 4$. What is the equation of the line? Use your answer to approximate $\tan(1.05)$. What is your percent error?

Problem 2: What is the equation of the line with slope $2x_0$ which passes through the point (x_0, x_0^2)?

Problem 3: The slope of the tangent line to $f(x) = x^2$ at the point (x_0, x_0^2) is $2x_0$. For what x_0 does the tangent line to $f(x)$ at (x_0, x_0^2) pass through the point $(0, -2)$?

Problem 4: If my position at time $t \geq 0$ is given by

$$s(t) = \frac{1}{4}\sqrt{t} + 1$$

what is my average rate of change on the intervals $[0, 1]$, $[1, 4]$, $[4, 9]$? (That's 3 different calculations you have to do.) Carefully explain how the graph of $s(t)$ shows me that as time progresses I'm slowing down. Is there an interval $[a, b]$ with $b > a$, such that my average speed on that interval is 0?

Problem 5: Let $g(x) = x^2 + x$. Calculate $g'(3)$ using a method similar to what we did in class (i.e. look at slopes of secant lines that are approaching the tangent line). No credit will be given for any other method you may have learned in some other class. (We'll get to those methods – I promise!)

Problem 6: What is the average rate of change of $q(x) = e^x + \ln(x)$ on the interval $[1, e]$?

Problem 7: Draw the graph of $f(x) = \cos(x)$ for $0 \leq x \leq 2\pi$. For what values of a is $f'(a) < 0$? For what values of a is $f'(a) = 0$? For what values of a is $f'(a) > 0$? Be sure to justify your answers.

Problem 8: In the month of August on the island of Numenor, there exist a certain number of elves and a certain number of mallorn trees. It turns out that the number of elves is three times the square root of the number of mallorn trees. As the number of mallorn trees changes the number of elves change. What are the units for the derivative of the number of elves with respect to the number of mallorn trees?

Problem 9: Acceleration is defined to be the derivative of velocity with respect to time. Velocity, in turn, is the derivative of position with respect to time. An elf falls out of a mallorn tree. At time t seconds the height of the elf is $h(t)$ feet. What are the units for the (downward) acceleration of the elf?

Problem 10: Read through sections 1.1 - 1.6 without spending too much time on them. Determine if there are any topics in those chapters that you would like to review. If so, either spend some time reviewing them on your own, or come see me and I'll be happy to go over those topics with you.

Problem 11: Do problems 15, 17, 19, and 21 on page 43 of the text. (These are algebra problems and don't require anything to do with the derivative.)