

MA 121: Homework 12 Additional Problems

Question 1: What major theorems are used in the proof that

$$\frac{d}{dt} \int_a^t f(x) dx = f(t)$$

where $f(x)$ is a continuous function?

Question 2: In the proof of FTC I, we encounter the following inequality (using the notation from class):

$$m_h \leq f(x) \leq M_h$$

for all x between t and $t + h$ (assuming $h > 0$). Explain what m_h and M_h are and how you know they exist.

Question 3: Starting with the inequality from Problem 2, show how to arrive at the inequality

$$m_h \leq \frac{1}{h} \int_t^{t+h} f(x) dx \leq M_h$$

with $h > 0$.

Question 4: Use the inequality from Problem (3) to explain why

$$\lim_{h \rightarrow 0^+} \frac{1}{h} \int_t^{t+h} f(x) dx = f(t).$$

Question 5: Explain the relevance of Problem (4) to the proof of FTC I.

Question 6: Use FTC I to prove FTC II (as we did in class.)

Question 7: Explain why it makes sense to define the average value of a function $f(x)$ on an interval $[a, b]$ to be

$$\frac{1}{b-a} \int_a^b f(x) dx.$$