Worksheet 6/26. Math 113 Summer 2014.

These problems are intended as supplementary material to the homework exercises and will hopefully give you some more practice with actual examples. In particular, they may be easier/harder than homework. Problems with an asterisk (*) should be more challenging than the rest.

1. Which of the following assignments are homomorphisms of groups

    (a) $f : \mathbb{Z} \to \mathbb{Z}$; $x \mapsto 2x$,

    (b) $f : \mathbb{Z} \to \mathbb{Z}$; $x \mapsto x + 1$,

    (c) $f : \mathbb{Z} \to \mathbb{Z}$; $x \mapsto \overline{x}$,

    (d) $f : W_3 \to W_4$; $A \mapsto A \mid$, where $A \mid$ is the wiring diagram with a vertical line between the last two vertices.

    (e) $f : GL_2(\mathbb{R}) \to GL_2(\mathbb{R})$; $A \mapsto A^3$,

    (f) $f : GL_2(\mathbb{R}) \to GL_2(\mathbb{R})$; $X \mapsto PX$, where $P = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$.

    (g) $f : \mathbb{Z}/3\mathbb{Z} \to \mathbb{Z}/3\mathbb{Z}$; $\overline{x} \mapsto \overline{x + 3}$

2. Let G be a finite group, $g \in G$ have order k. Check that the homomorphism

$$f : \mathbb{Z}/k\mathbb{Z} \to < g > \; ; \; \overline{i} \mapsto g^i,$$

    is well-defined. Show that it is an isomorphism.

3. Prove Lemma 4.1.2.

4. (a) Are there any nontrivial homomorphisms $\mathbb{Z}/2\mathbb{Z} \to \mathbb{Z}/5\mathbb{Z}$?

    (b) Are there any nontrivial homomorphisms $\mathbb{Z}/5\mathbb{Z} \to \mathbb{Z}/2\mathbb{Z}$?

5. Find cosets of $H = \{\pm 1\} \subset \mathcal{Q}_8$, where $\mathcal{Q}_8$ is the quaternion group.

6. Determine the cosets of $H = \{\overline{0}, \overline{3}, \overline{6}\} \subset \mathbb{Z}/9\mathbb{Z}$.

7. * Find $r \in W_5$ of order 6, $s \in W_5$ of order 2 such that $srs = r^{-1}$. Using this, define an injective homomorphism $f : D_{12} \to W_5$. Explain why there exists a subgroup of S_5 isomorphic to D_{12}