Worksheet 6/25. Math 113 Summer 2014.

These problems are intended as supplementary material to the homework exercises and will hopefully give you some more practice with actual examples. In particular, they may be easier/harder than homework. Problems with an asterisk (*) should be more challenging than the rest.

1. Prove that the subgroup of a group G generated by an element $g \in G$ is actually a subgroup. Prove furthermore that it's abelian.
2. Prove that If every element of g of a group G satisfies $g^2 = e$, then G is abelian.
3. Find an abelian subgroup of D_8 (remember, though, that D_8 itself is not abelian).
4. Find an element of order 1, 2, 3, and 4 in the wiring group W_4. Are there any elements with higher orders?
5. Does $\mathbb{Z}/5\mathbb{Z}$ have any proper nontrivial subgroups? Why or why not?
6. The vector space R^2 is a group under addition[1]. Is the subset $\{(x, y) \mid x = 0 \text{ or } y = 0\}$ a subgroup?
7. * Find the order of the group generated by elements x, y subject only to the relations $x^4 = y^2 = (xy)^2 = e$.

[1] In fact, every vector space is an abelian group under its addition operation.