Worksheet 7/3. Math 113 Summer 2014.

These problems are intended as supplementary material to the homework exercises and will hopefully give you some more practice with actual examples. In particular, they may be easier/harder than homework. Problems with an asterisk (*) should be more challenging than the rest.

1. (a) Prove that $o(\sigma) = 3$ if and only if $\sigma \in C((124))$.
   (b) List all the elements in the conjugacy class $C((124))$ in S_4.
   (c) Determine the centraliser $\mathrm{Cent}_{S_4}((124))$.

2. Prove that if G acts on a set X, then x is a fixed point of this action if and only if $\mathrm{Stab}_G(x) = G$.

3. For your convenience, here is a list of all the subgroups of D_8: D_8 itself, $\{e, r, r^2, r^3\}$, $\{e, s, r^2, rs^2\}$, $\{e, rs, r^2, r^3s\}$, $\{e, r^2\}$, $\{e, rs\}$, $\{e, r^2s\}$, $\{e, r^3s\}$, and the trivial subgroup $\{e\}$. In total there are 10 subgroups. Let D_8 act on $\mathrm{Sub}(D_8)$ by conjugation. Find all the orbits of this action. Use this to determine which subgroups are normal.

4. Let G be a group of order 6.
   (a) Show that G admits at least three conjugacy classes, and no more than six.
   (b) Prove that $Z(G)$ is the union of all $g \in G$ such that $C(g) = \{g\}$. (This holds for an arbitrary group, not just G with $|G| = 6$.)
   (c) Show that G must admit either 3 or 6 conjugacy classes.

5. * (This is a continuation of Problem 4) Prove that if G has order 6 and admits 6 conjugacy classes then G is cyclic. Prove that if G has order 6 and admits 3 conjugacy classes then $G \cong S_3$.

   You have just shown that every group of order 6 *is isomorphic to either* $\mathbb{Z}/6\mathbb{Z}$ *or* S_3.