# Worksheet 7/14. Math 113 Summer 2014.

*These problems are intended as supplementary material to the homework exercises and will hopefully give you some more practice with actual examples. In particular, they may be easier/harder than homework. Problems with an asterisk (*) should be more challenging than the rest.*

1. Describe (up to isomorphism) all abelian groups of order 16.

2. Describe (up to isomorphism) all abelian groups of order 72.

3. A certain abelian group $G$ has order 360, and contains an element of order 4 and an element of order 9. What are the possible isomorphism classes of $G$?

4. (a) Show that $G = \mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z}$ contains subgroups $\{(0,0,0)\} = G_0 \subset G_1 \subset G_2 \subset G_3 = G$ such that each quotient $G_i/G_{i-1}$ $(i = 1, 2, 3)$ is cyclic.

   (b) Prove that an abelian group $G$ of order $p^r$ contains subgroups $\{e\} = G_0 \subseteq G_1 \subseteq \cdots \subseteq G_r = G$ such that each successive quotient $G_i/G_{i-1}$ $(i = 1, \cdots, r)$ is cyclic.

   (c) Let $G$ be a finite abelian group of order $n$. Prove by induction on the number of distinct primes dividing $n$ that $G$ contains subgroups $\{e\} = G_0 \subseteq G_1 \subseteq \cdots \subseteq G_m = G$ such that each successive quotient $G_i/G_{i-1}$ $(i = 1, \cdots, r)$ is cyclic.

5. * Prove that if $G$ is a finite abelian group (written multiplicatively) of order $n$ such that, for each $m \mid n$, there are exactly $m$ elements $g \in G$ such that $g^m = e$, then $G$ is cyclic.