Worksheet 7/1. Math 113 Summer 2014.

These problems are intended as supplementary material to the homework exercises and will hopefully give you some more practice with actual examples. In particular, they may be easier/harder than homework.

1. Let C be the numbers appearing on the clock face. Give three distinct actions of $\mathbb{Z}/2\mathbb{Z}$ on C.

2. Prove that there are no nontrivial actions of $\mathbb{Z}/5\mathbb{Z}$ on $\{1, 2, 3, 4\}$.[1]

3. Let $G = \mathrm{GL}_2(\mathbb{R})$, $S = \mathbb{R}^2$. Prove that

$$g \cdot v \stackrel{def}{=} gv, \quad g \in G, v \in S,$$

defines an action of G on S. What is the number of orbits of this action? What is $\mathrm{Stab}_G(e_1)$, where e_1 is the standard basis vector with a 1 in the first entry, 0 in the second entry.

4. Consider D_8 acting on $\mathrm{Sub}(D_8)$ by conjugation. What is the orbit $\mathcal{O}_H$ of $H = \{e, r^2\}$? What is $\mathrm{Stab}_{D_8}(H)$? What do you notice about $|\mathcal{O}_H|$ and $|\mathrm{Stab}_{D_8}(H)|$?

5. * Let $G = \mathrm{GL}_2(\mathbb{R})$, $B = \{\text{upper triangular matrices in } G\}$. Prove that B acts on the set G/B, the set of left cosets of B in G, via

$$b \cdot (gB) \stackrel{def}{=} bgB, \quad b \in B, gB \in G/B,$$

and that there are exactly two distinct orbits - one containing the identity matrix, the other containing $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$.

[1] An action of G on S is called **trivial** if $g \cdot x = x$, for every $g \in G$, $x \in S$.