Worksheet August 11th. Math 113 Summer 2014.

These problems are intended as supplementary material to the homework exercises and will hopefully give you some more practice with actual examples. In particular, they may be easier/harder than homework. Problems with an asterisk (*) should be more challenging than the rest.

1. Which of the following extensions are normal?

   (a) $\mathbb{Q} \subset \mathbb{Q}(i)$
   (b) $\mathbb{Q} \subset \mathbb{Q}(\sqrt[5]{2})$
   (c) $\mathbb{Q}(\sqrt{3}) \subset \mathbb{Q}(\sqrt[4]{3})$

2. Compute the Galois group $G = \mathrm{Gal}(\mathbb{Q}(\sqrt{5}, \sqrt{7}) : \mathbb{Q})$. Find all the subgroups of G by identifying it with a familiar group. Use this and the Galois correspondence to list all intermediate subfields of the extension $\mathbb{Q} \subset \mathbb{Q}(\sqrt{5}, \sqrt{7})$.

3. Prove that if $K \subset F \subset L$, and L is normal over K, then L is normal over F as well.

4. Prove that the extension $\mathbb{Q} \subset \mathbb{Q}(e^{2\pi i/6})$ has no intermediate subfields except $\mathbb{Q}$ and itself.

5. Use the computation of $\mathrm{Gal}(\mathbb{Q}(i, \sqrt[4]{2}) : \mathbb{Q})$ from lecture to conclude that there is only one degree four normal subextension $\mathbb{Q} \subset F \subset \mathbb{Q}(i, \sqrt[4]{2})$.