

Linear Algebra (Math 110), Summer 2012

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(July 11, 2012 corrected version)

Abstract

These are notes for the upper division course 'Linear Algebra' (Math 110) taught at the University of California, Berkeley, during the summer session 2012. Students are assumed to have attended a first course in linear algebra (equivalent to UCB Math 54). The aim of this course is to provide an introduction to the study of finite dimensional vector spaces over fields of characteristic zero and linear morphisms between them and to provide an abstract understanding of several key concepts previously encountered. The main topics to be covered are: basics of vector spaces and linear morphisms, the Jordan canonical form and Euclidean/Hermitian spaces.

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