Math 110, Summer 2012 : Bilinear forms review problems

Basics

1. Determine which of the following bilinear forms are symmetric/antisymmetric/neither, non-degenerate:

- $B_A \in \mathrm{Bil}_{\mathbb{Q}}(\mathbb{Q}^3)$, where $A = \begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & -1 \\ -1 & 1 & 2 \end{bmatrix}$,

- $B_A \in \mathrm{Bil}_{\mathbb{R}}(\mathbb{R}^4)$, where $A = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & -1 & 0 & 2 \\ 0 & 0 & 0 & 1 \\ 1 & 2 & 1 & 5 \end{bmatrix}$,

- $B : Mat_2(\mathbb{R}) \times Mat_2(\mathbb{R}) \to \mathbb{R}\ ;\ (A, B) \mapsto \mathrm{tr}(A^t X B)$, where $X = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$.

- $B : \mathbb{R}^4 \times \mathbb{R}^4 \to \mathbb{R}\ ;\ (\underline{u}, \underline{v}) \mapsto u_1v_2 + u_2v_1 + u_1v_4 + u_4v_1 + u_2v_2 + u_2v_4 + u_4v_2 + u_4v_4 + 2u_3v_3$.

- $B : Mat_2(\mathbb{R}) \times Mat_2(\mathbb{R}) \to \mathbb{R}\ ;\ (A, B) \mapsto \mathrm{tr}(A^t X B)$, where $X = \begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix}$.

Canonical forms of symmetric nondegenerate real bilinear forms

Determine the canonical form of the following real symmetric nondegenerate bilinear forms.

- $B_A \in \mathrm{Bil}_{\mathbb{R}}(\mathbb{R}^3)$, where $A = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 3 & 2 \\ 0 & 2 & -1 \end{bmatrix}$,

- $B_A \in \mathrm{Bil}_{\mathbb{R}}(\mathbb{R}^3)$, where $A = \begin{bmatrix} -3 & 2 & -2 \\ 2 & 1 & 0 \\ -2 & 0 & 5 \end{bmatrix}$,

- $B_A \in \mathrm{Bil}_{\mathbb{R}}(\mathbb{R}^4)$, where $A = \begin{bmatrix} 0 & 1 & 2 & -1 \\ 1 & 0 & -3 & 1 \\ 2 & -3 & 1 & 0 \\ -1 & 1 & 0 & 5 \end{bmatrix}$.

Which of the bilinear forms are inner products? For those that are not inner products determine $\underline{x}$ such that $B_A(\underline{x}, \underline{x}) < 0$.

Projections, Gram-Schmidt

Determine $\mathrm{proj}_U v$, for the given subspace $U \subset V$ and $v \in V$. You will need to determine an orthogonal basis of U using Gram-Schmidt (with respect to the 'dot product').

- $U = \ker T_A$, where $A = \begin{bmatrix} 1 & -3 & 1 \\ 3 & -1 & 0 \end{bmatrix}$, $v = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$.

- $U = \mathrm{im}\, T_A$, where $A = \begin{bmatrix} -1 & 0 & 2 & -1 \\ 2 & 0 & 2 & 1 \\ 0 & 0 & -1 & 2 \end{bmatrix}$, $v = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$,

- $U = \left\{ \begin{bmatrix} 1 \\ -1 \\ 2 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix} \right\}, v = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}.$

Determine $U^{\perp}$ for the above subspaces U.