Worksheet 7/26. Math 110, Summer 2012

*An asterisk * denotes a harder problem. Speak to your neighbours, these problems should be discussed.*

Canonical form of real symmetric nondegenerate bilinear forms

Consider the following (nondegenerate, symmetric) bilinear forms

- $B_1 : \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{R}\ ;\ (u, v) \mapsto u_1 v_3 + 5u_2 v_2 - 6u_3 v_1$.
- $B_2 : Mat_2(\mathbb{R}) \times Mat_2(\mathbb{R}) \to \mathbb{R}\ ;\ (X, Y) \mapsto \mathrm{tr}(XY)$.

1. Determine bases $\mathcal{B}_1, \mathcal{B}_2$ such that

$$[B_i]_{\mathcal{B}_i} = \begin{bmatrix} d_1 & & \\ & \ddots & \\ & & d_n \end{bmatrix}, \quad d_i \in \{1, -1\}.$$

2.Consider the (nondegenerate symmetric) bilinear forms

- $B_1 : \mathbb{Q}^3 \times \mathbb{Q}^3 \to \mathbb{Q}\ ;\ (u, v) \mapsto u_1 v_3 + 5u_2 v_2 - 6u_3 v_1$.
- $B_2 : Mat_2(\mathbb{Q}) \times Mat_2(\mathbb{Q}) \to \mathbb{Q}\ ;\ (X, Y) \mapsto \mathrm{tr}(XY)$.

Is it possible to find base $\mathcal{C}_1, \mathcal{C}_2$ such that $[B_i]_{\mathcal{C}_i}$ is *diagonal*?