Worksheet 7/11. Math 110, Summer 2012

*An asterisk * denotes a harder problem. Speak to your neighbours, these problems should be discussed.*

Invariant subspaces

1. Consider the matrix

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Show that

$$U = \left\{ \underline{x} \in \mathbb{C}^4 \mid x_1 + x_2 + x_3 + x_4 = 0 \right\} \subset \mathbb{C}^4,$$

is A-invariant. Determine the eigenvalues of A (you should have seen that 1 is an eigenvalue of A). Determine the 1-eigenspace E_1. Find an A-invariant direct sum complement of U.

2. Provide a criterion for determining when a $n \times n$ matrix A is diagonalisable using A-invariant subspaces of $\mathbb{C}^n$.

(*Your criterion should be of the form, A is diagonalisable if and only if $\mathcal{P}$, where $\mathcal{P}$ is some property of $\mathbb{C}^n$ related to A-invariant subspaces.*

Nilpotent endomorphisms

4. Show that the following matrices are nilpotent and determine an invertible matrix P such that $P^{-1}AP$ is a block diagonal matrix consisting of 0-Jordan blocks of nonincreasing size as you look from left to right. Determine the partition associated to A, $\pi(A)$.

1. $A = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$.

2. $A = 0_3 \in Mat_3(\mathbb{C})$, the zero matrix.

3. $A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}$.

4. $A = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

5. $A = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

Are the matrices in 4, 5 similar? Justify your answer.