Worksheet 7/3. Math 110, Summer 2012

*An asterisk * denotes a harder problem. Speak to your neighbours, these problems should be discussed.*

Elementary matrices

1. Determine the $4\times$ elementary matrices corresponding to the elementary row/column operations

- swap row 2 with row j
- multiply row 3 by $\frac{1}{2}$
- swap column 2 with column 3
- add 2 times row 1 to row 3
- add -3 times column 4 to column 1.

2. Row/column reduce the following matrices until they take the form

$$\begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}.$$

$$A = \begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix},$$

$$B = \begin{bmatrix} 1 & 1 & 2 \\ 2 & -1 & 3 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 0 \\ -1 & 1 & 1 \end{bmatrix}.$$

Now use elementary matrices to determine Q, P such that

$$Q^{-1}AP = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}.$$

Can you determine, using only Q and P, bases for $\ker T_A$ and $\operatorname{im} T_A$? Also, can you determine a basis of a direct sum complements of $\ker T_A$, $\operatorname{im} T_A$?

Eigenstuff

3. Determine the characteristic polynomial, eigenvalues, bases for eigenspaces for the following matrices

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \quad \begin{bmatrix} 2 & -1 \\ 2 & 1 \end{bmatrix}, \quad \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix},$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}, \quad \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{bmatrix}.$$

What are the algebraic/geometric multiplicities of each eigenvalue you have found?