Worksheet 6/28. Math 110, Summer 2012

*An asterisk * denotes a harder problem. Speak to your neighbours, these problems should be discussed.*

Finding bases

0. Consider the following ordered bases of $\mathbb{Q}^3$

$$\mathcal{S}^{(3)},\ \mathcal{B}_1 = \left\{ \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\},\ \mathcal{B}_2 = \left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \right\}.$$

Given a linear morphism $f : \mathbb{Q}^3 \to \mathbb{Q}^3$ it is usually quite easy to determine $[f]_{\mathcal{S}^{(3)}}$. How can you determine $[f]_{\mathcal{B}_1}^{\mathcal{B}_2}$ assuming you know $[f]_{\mathcal{S}^{(3)}}$? What about $[f]_{\mathcal{B}_2}^{\mathcal{B}_1}$

Use your method to determine $[f]_{\mathcal{B}_1}^{\mathcal{B}_2}$ and $[f]_{\mathcal{B}_2}^{\mathcal{B}_1}$, for

$$f : \mathbb{Q}^3 \to \mathbb{Q}^3 \ ; \ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \mapsto \begin{bmatrix} -x_1 + 2x_3 \\ 2x_1 + x_2 + x_3 \\ x_2 - x_3 \end{bmatrix}.$$

Consider the following bases of $Mat_2(\mathbb{Q})$

$\mathcal{S} = (e_{11}, e_{12}, e_{21}, e_{22})$, $\mathcal{B}_1 = (e_{11}, e_{12} - e_{21}, e_{22}, e_{12} + e_{21})$, $\mathcal{B}_2 = (e_{11} + e_{22}, e_{21}, e_{12}, e_{11} - e_{22})$.

Consider the linear morphism

$$g : Mat_2(\mathbb{Q}) \to Mat_2(\mathbb{Q}) \ ; \ A \mapsto A + A^t,$$

where A^t is the transpose of A. Determine $[g]_{\mathcal{S}}$ and use this to determine $[g]_{\mathcal{B}_1}$ and $[g]_{\mathcal{B}_2}$.

(*This will require to compute the inverse of* 4×4 *matrices! Recall that to compute the inverse of an invertible* $n \times n$ *matrix* B *you form the* $n \times 2n$ *matrix* $[B\ I_n]$ *and then row-reduce this to obtain* $[I_n\ B^{-1}]$.)

1. In groups think about how to prove Theorem 1.7.3.

(*You will need to use the* <u>*uniqueness*</u> *property of* $[f]_{\mathcal{B}}^{\mathcal{C}}$ *here.*)

2. In groups think about how to prove Theorem 1.7.4.

(*You should proceed in a similar manner as I did during class in proving the same statement for the standard matrix* A_f.)