Worksheet 6/20. Math 110, Summer 2012

*An asterisk * denotes a harder problem. Speak to your neighbours, these problems should be discussed.*

Sum, direct sum

1. In $\mathbb{R}^2$ give examples of subspaces $U, W \subset \mathbb{R}^2$ such that $U + W = \mathbb{R}^2$. Is your example a *direct sum* (ie, is $U \cap W = \{0_{\mathbb{R}^2}\}$?).

Is it possible to find subspaces $U, W \subset \mathbb{R}^2$ such that $U + W = \mathbb{R}^2$ but this sum is not a direct sum?

2. Do the same thing for $\mathbb{R}^3$.

Linear independence

1. Explain why V is always linearly dependent, if V is a $\mathbb{K}$-vector space. If $U \subset V$ is a vector subspace is it true that U is linearly dependent? Explain your answer.

2. Consider the $\mathbb{Q}$-vector space $\mathbb{Q}^{\mathbb{N}} = \{f : \mathbb{N} \to \mathbb{Q}\}$. For $f \in \mathbb{Q}^{\mathbb{N}}$ we can denote f as an infinite sequence

$$f \equiv (f(1), f(2), f(3), f(4), ...).$$

Let $e_i \in \mathbb{Q}^{\mathbb{N}}$, $i \in \mathbb{N}$, denote the functions $e_i : \mathbb{N} \to \mathbb{Q}$, with $e_i(j) = 0$, if $i \neq j$, and $e_i(i) = 1$. So, in the above notation, e_i is the infinite sequence with 0 everywhere except a 1 in the i^{th} entry.

Explain why $E = \{e_i \mid i \in \mathbb{N}\} \cup \{(1, 1, 1, 1, ...)\}$ is *linearly independent*. (*You can either show this directly or use a theorem from the notes to help you.*)

3. Let $E \subset \mathbb{C}^n$ be a finite nonempty subset. Write down an algorithm[1] to determine the linear (in)dependence of E.

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[1] 'Algorithm' = 'recipe'.