Worksheet 6/18. Math 110, Summer 2012

An asterisk * denotes a harder problem. Speak to your neighbours, these problems should be discussed.

Sets, functions

Get yourselves into groups of 4-5. Denote your group of students by G, or whatever. Find functions $f : G \to G$ that are injective, surjective and bijective.

Fields

Is it possible to row-reduce the following matrices to echelon form using only $\mathbb{Q}$-scalars? What about reduced echelon form?

$$A = \begin{bmatrix} 1 & -7 \\ -350/12 & 14 \end{bmatrix}, \quad B = \begin{bmatrix} \sqrt{2} & 1 \\ -1 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 4 & \sqrt{2} \\ 1 & 2\sqrt{2} \end{bmatrix}.$$

Is it possible to row-reduce these matrices to reduced echelon form using $\mathbb{Q}(\sqrt{2})$-scalars?

Vector spaces, subspaces

1. Rewrite Axioms VS1-VS8 by replacing the functions α, σ by the usual notations '$+$' and '$\cdot$'.

2. Consider the $\mathbb{K}$-vector space (Z, α, σ) where Z contains exactly one element. Why is it true that there can only exist one $\mathbb{K}$-vector space structure on Z, ie, if (Z, β, τ) is another $\mathbb{K}$-vector space, then $\alpha = \beta, \sigma = \tau$.

3. Which of the following subsets U are subspaces of the given $\mathbb{K}$-vector space V? Verify your answer.

$$a) \quad U = \{\underline{x} \in \mathbb{R}^2 \mid 2x_1 - x_2 = 0\} \subset \mathbb{R}^2 = V,$$

$$b) \quad U = \left\{A \in Mat_{2,2}(\mathbb{Q}) \mid A = -A^t\right\} \subset Mat_{2,2}(\mathbb{Q}) = V, \quad \text{where } A^t \text{ is the } \textit{transpose} \text{ of } A,$$

$$c)^* \quad U = \{f \in \mathbb{R}^{\{1,2,3\}} \mid f(1) = f(2) = f(3)\} \subset \mathbb{R}^{\{1,2,3\}} = V.$$

For c): it will might help to rewrite the property defining U.