Math 110, Summer 2012 Short Homework 8

Due Monday 7/23, 10.10am, in Etcheverry 3109. Late homework will not be accepted.

0. Was this homework assignment too easy/too difficult/about right? Any other comments are welcome.

Calculations

1. Consider the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}.$$

i) Determine $\chi_A(t)$.

ii) Show that A is NOT diagonalisable without appealing to algebraic/geometric multiplicities. (*Use the diagonalisablity criterion involving the minimal polynomial. What must the minimal polynomial be if A were diagonalisable?*

iii) Determine the subspaces

$$U_1 = \ker T_{(A-2I_4)^3}, \; U_2 = \ker T_A,$$

and a basis $\mathcal{C} = (c_1, c_2, c_3) \subset U_1$.

iv) Consider the endomorphism

$$f : U_1 \to U_1 \; ; \; u \mapsto Au - 2u.$$

Determine $B = [f]_{\mathcal{C}}$.

v) Show that B is nilpotent and find a basis $\mathcal{C}' \subset U_1$ such that $[f]_{\mathcal{C}'}$ is block diagonal, each block being a 0-Jordan block.

vi) Determine an invertible matrix $P \in GL_4(\mathbb{C})$ such that

$$P^{-1}AP = J,$$

where J is the Jordan form of A.

Proofs

2. Let $f \in \mathrm{End}_{\mathbb{C}}(V)$ be such that $\chi_f(t) = t^{\dim V}$. Prove that f is nilpotent.

3. Let $A \in Mat_7(\mathbb{C})$ be an invertible matrix and such that

$$A^7 - 6A^4 - 6A^6 + 11A^5 = 0_7 \in Mat_7(\mathbb{C}).$$

Prove that A is diagonalisable. (*Hint: It may be useful to know that* 1 *is an eigenvalue of A. You will need to perform the division algorithm (ie, long division of polynomials) at some point in your solution.*)