Math 110, Summer 2012 Short Homework 11

Due Thursday 8/2, 10.10am, in Etcheverry 3109. Late homework will not be accepted.

0. Was this homework assignment too easy/too difficult/about right? Any other comments are welcome.

Calculations

1. Show that the following bilinear form is an inner product on $\mathbb{R}^3$,

$$B : \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{R} \; ; \; (\underline{x}, \underline{y}) \mapsto x_1y_1 + 2x_2y_2 + 3x_3y_3 - x_1y_2 - x_2y_1 - x_2y_3 - x_3y_2.$$

(*You must show that B is symmetric, nondegenerate and positive definite.*)

Determine an Euclidean isomorphism

$$f : (\mathbb{R}^3, B) \to \mathbb{E}^3.$$

What is the length of $\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$, with respect to B?

2. Using the inner product B above, determine the orthogonal complement of $\text{span}_{\mathbb{R}}\{e_1\} \subset \mathbb{R}^3$ (with respect to B).

3. Show that the following bilinear form is NOT and inner product

$$B' : \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{R} \; ; \; (\underline{x}, \underline{y}) \mapsto x_1y_1 + x_2y_3 + x_3y_2,$$

by finding a vector $\underline{x}_0 \in \mathbb{R}^3$ such that $B'(\underline{x}_0, \underline{x}_0) < 0$.

(*Hint: determine the canonical form of B'.*)

Proofs

4. Prove Pythagoras' theorem (Theorem 3.3.6).

5. Prove the Cauchy-Schwarz inequality (Theorem 3.3.6) as follows: let $u, v \in V$.

- if $v = 0_V$ then the result is easy (you must still show this!).
- if $v \neq 0_V$ then consider

$$\langle u - \lambda v, u - \lambda v \rangle, \text{ for any } \lambda \in \mathbb{R}.$$

  By making an informed choice of λ (expand out the above expression) you will obtain

$$\langle u, u \rangle \langle v, v \rangle \geq \langle u, v \rangle^2.$$

  Use this to deduce the result.

6. Prove that an Euclidean morphism $f : (V_1, \langle , \rangle_1) \to (V_2, \langle , \rangle_2)$ is injective.

7. Let $(V, \langle , \rangle)$ be an Euclidean space, $S \subset V$ a nonempty subset. Prove that $S^\perp$ is a subspace and that

$$(\text{span}_{\mathbb{R}} S)^\perp = S^\perp.$$

(*To show two sets A, B are equal, it suffices to show that $A \subset B$ and $B \subset A$.*)