

LONG HOMEWORK 4 : MATH 110 . SUMMER 2012
(SOME TIPS)

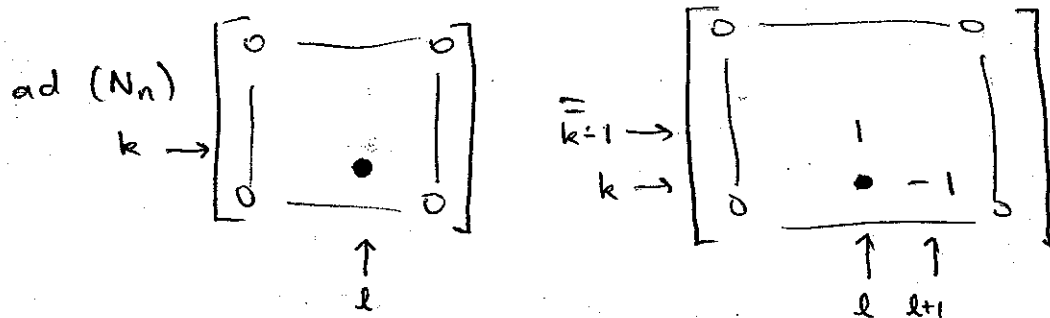
b) TO DETERMINE (N_π) , π PARTITION OF 3, LET

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \quad \& \quad \text{CONSIDER} \quad AN_\pi = N_\pi A.$$

c) THE RESULT

$$\text{ad}(N_n)(e_{kl}) = \begin{cases} -e_{k,l+1} & , k=1, 1 \leq l < n \\ e_{k-1,l} & , 2 \leq k \leq n, l=n \\ e_{k-1,l} - e_{k,l+1} & , 2 \leq k \leq n, 1 \leq l < n \\ 0 & k=1, l=n \end{cases}$$

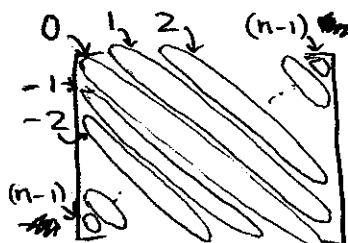
CAN BE REPRESENTED PICTORIALLY:



SO $\text{ad}(N_n)(e_{kl})$ IS MATRIX WITH 1 ABOVE kl -ENTRY,
& -1 TO THE RIGHT OF kl -ENTRY.

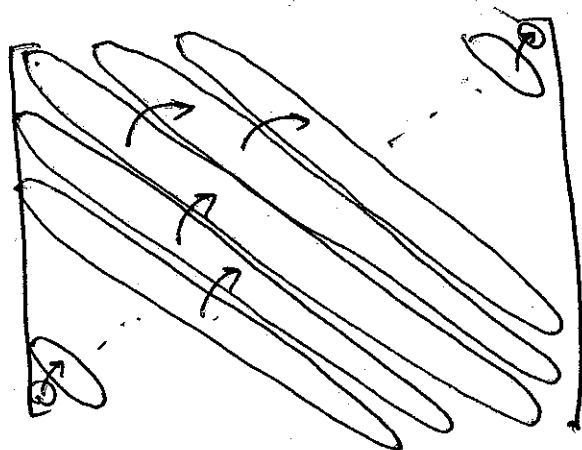
IN PARTICULAR, ON TOP ROW (i.e. $k=1$) WE JUST GET A
-1 TO THE RIGHT OF kl -ENTRY; ON RIGHTMOST COLUMN,
JUST GET A 1 ABOVE kl -ENTRY.

THINKING IN THIS MANNER YOU CAN CONVINCED YOURSELF
THAT, IF WE LABEL THE DIAGONALS OF AN ARBITRARY $n \times n$
MATRIX AS



SO, E.G. DIAGONAL
0 IS MAIN DIAGONAL

THEN, $\text{ad}(N_n)$ MOVES ENTRIES IN DIAGONAL i TO ENTRIES IN DIAGONAL $i+1$



→ = ACTION OF $\text{ad}(N_n)$ ON MATRIX WITH ONLY NONZERO ENTRIES IN DIAGONAL i .

• c) ii) IS ASKING YOU TO SHOW THAT

DIAGONAL i IS MAPPED ONTO DIAGONAL $(i+1)$
(FOR $i=0, \dots, n-2$) BY $\text{ad}(N_n)$

• c) iii) IS ASKING YOU TO SHOW THAT THE RESTRICTION OF $\text{ad}(N_n)$ TO DIAGONAL i ($i=-1, \dots, -(n-1)$) IS INJECTIVE (RECALL, IF f INJECTIVE THEN f TAKES L.I. SETS TO L.I. SETS)