Math 284 - Canonical Bases in Representation Theory.
Spring 2017, TTh 1-2.30pm. Science Center 411.

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Office hours: Thursday 3-4pm
Course website: `http://math.harvard.edu/~gmelvin/current.html`
   Announcements, handouts and notes can be found at the website.

Course description: This course will focus on some of the topics and results in representation theory and combinatorics that resulted in the discovery of cluster algebras by Fomin-Zelevinsky in the late 90s. To soothe our digestion we will mainly focus on type A.

Tentative Schedule:

Weeks 1-2 Introduction & overview. Basics of representation theory. 'Good bases' of Gelfand-Zelevinsky.

Weeks 2-3 Quantum groups, canonical bases.

Weeks 3-6 Kashiwara crystals. Character formulae.

Weeks 6-7 Relation to polyhedral combinatorics. Total positivity.

Weeks 8-9 Results of Berenstein-Zelevinsky.

Weeks 9-11 Berenstein-Kazhdan geometric crystals. Relations to mirror constructions.

Weeks 11-12 Spillover. Additional topics.

Prerequisites: Some representation theory and Lie theory; a course in algebraic geometry (quasi-projective varieties will suffice). Or a willingness to believe that undergraduate linear algebra is 'just the tip of the iceberg'.

Resources: A (growing) list of papers will be updated online. For the material on crystals there is a fantastic new book by Bump-Schilling, '*Crystal Bases: Representations and Combinatorics*' which I highly recommend. Technically, it has not yet been published but I think we can get around this... Let me know if you are interested in obtaining a copy.

Grading: If you need a grade: you must TeX up notes for at least one lecture and submit a final paper.

Homework: Problem sets will be made available periodically. We will make use of the (open) problems that appear in Bump-Schilling's new book.

Final paper: Submit a 8-15 page expository paper on a topic that is related to the course. Final papers are due 5/4.

Important dates:
   2/20: University holiday, no class.
   2/21: Fifth 'Monday'
   3/11-3/19: Spring Recess
   4/26: Last day of instruction
   5/4: Final paper submission