Middlebury College

Multivariable Calculus: Spring 2018

Homework

Due March 12, 6pm

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Some thoughts and advice:

- You should expect to spend at least $1-2$ hours on problem sets. A lot of practice problem-solving is essential to understand the material and skills covered in class. Be organised and do not leave problem sets until the last-minute. Instead, get a good start on the problems as soon as possible.
- When approaching a problem think about the following: *do you understand the words used to state the problem? what is the problem asking you to do? can you restate the problem in your own words? have you seen a similar problem worked out in class? is there a similar problem worked out in the textbook? what results/skills did you see in class that might be related to the problem?*

  If you are stuck for inspiration, use the course `piazza` forum (accessible via the course Canvas site). However, don't just ask for the solution - provide your thought process, the difficulties you are having, and ask a coherent question in complete English sentences. Remember the 3RA approach to asking questions outlined in the course syllabus.
- Form study groups - get together and work through problem sets together. **This will make your life easier!** However, you must write your solutions *on your own* and *in your own words*.

⋆ ⋆ ⋆ You **are not allowed** to use any additional resources (e.g. solution manuals, `stackoverflow` etc). If you are concerned then please ask.

- The problems in parentheses are for extra practice and optional (in particular, they do not need to be submitted). **Problems for submission from the textbook are underlined. Additional problems not from the textbook must be submitted**.

  To gain mastery of a topic you should expect to attempt a significant proportion of the problems in the textbook ($> 60\%$(!)).
- Answers to odd-numbered exercises are at the back of the textbook. However, you need to submit a worked solution and provide justification for how you determined the answer.

- **Read/recap:** §2.1
- **Problems:** §$X.y$ refers to Chaper X, Section y of *Vector Calculus*, by Colley (4th Edition). All problems are taken from the 4th Edition.

  §2.1: (1), 2, (4), 5, (6), 7, (8), 10, 11, 12.

Problem A: Consider the plane $\Pi: 3x - y + 2z = -1$. Define the function

$$\begin{aligned} d: \mathbb{R}^3 &\to \mathbb{R} \\ P &\mapsto (\text{distance from } P \text{ to } \Pi)^2 \end{aligned}$$

1. For $P = (x, y, z)$, determine $d(x, y, z)$.
2. What is the range of d?
3. Explain why d is not injective.

4. Explain why d is not surjective.

Problem B: Define the function

$$\begin{array}{rcl} \underline{f} : \mathbb{R}^2 & \to & \mathbb{R}^2 \\ P & \mapsto & (\text{distance from } P \text{ to } (1,0), \text{distance from } P \text{ to } (0,1)) \end{array}$$

1. Determine the component functions $\underline{f}_1(x,y)$, $\underline{f}_2(x,y)$ of $\underline{f}$.

2. For a function $f : X \to Y$, the **preimage of** $y \in Y$ is the set of inputs $x \in X$ such that $f(x) = y$ (i.e. all solutions to the equation $f(x) = y$). Describe geometrically the preimage under $\underline{f}$ of $(1,1)$. Explain why your solution implies $\underline{f}$ is not injective.

3. Show that $\underline{f}$ is not surjective by finding a point $Q \in \mathbb{R}^2$ which is not in the range of $\underline{f}$.