SOLUTIONS TO ADDITIONAL PROBLEMS : MATH 223

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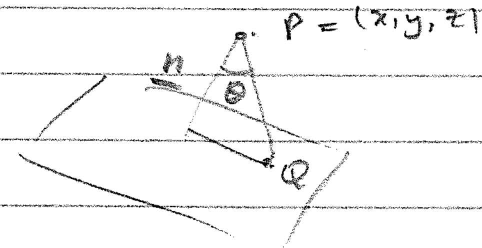
Problem A : Let $Q = (0,1,0)$

distance from P to Π

$$= |\cos\theta| \cdot |\overrightarrow{PQ}|$$



$$= \frac{|\underline{n} \cdot \overrightarrow{PQ}|}{|\underline{n}|},$$

where $\underline{n} = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$.

$$\Rightarrow \left| \frac{\underline{n} \cdot (x, y-1, z)}{\sqrt{14}} \right|^2 = \frac{1}{14}\left(3x - (y-1) + 2z\right)^2.$$

a) $d(x,y,z) = \frac{1}{14}\left(3x - (y-1) + 2z\right)^2$

b) $\mathbb{R}_{\geq 0}$

c) All points on Π satisfy $d = 0$.

d) There is no $P \in \mathbb{R}^3$ s.t. $d(P) = -1$.

Problem B:

Let $(x,y) = P$. Then

distance from P to $(1,0)$
$= \sqrt{(x-1)^2 + y^2}$

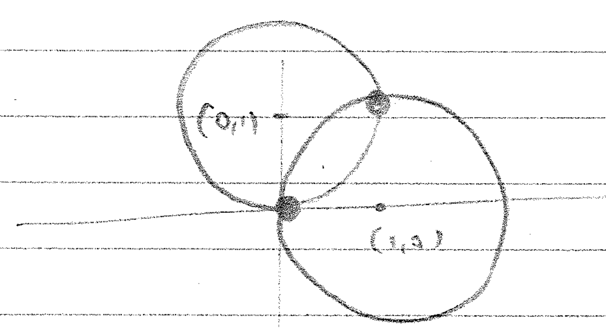
distance from P to $(0,1)$
$= \sqrt{x^2 + (y-1)^2}$

a) $f_1(x,y) = \sqrt{(x-1)^2 + y^2}$

$f_2(x,y) = \sqrt{x^2 + (y-1)^2}$

b) This is all points s.t.

distance to $(1,0)$
$=$ distance to $(0,1)$
$= 1$



i.e. $(0,0)$ and $(1,1)$.
we have $\underline{f}(0,0) = \underline{f}(1,1) \Rightarrow \underline{f}$ not injective

3) $(-1,-1)$ is not in range of f

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Problem A:

Using FToC:

$$\frac{\partial f}{\partial x} = e^{x^2}$$

$$\cancel{\frac{\partial g}{\partial}} \quad \frac{\partial f}{\partial y} = -e^{y^2}.$$

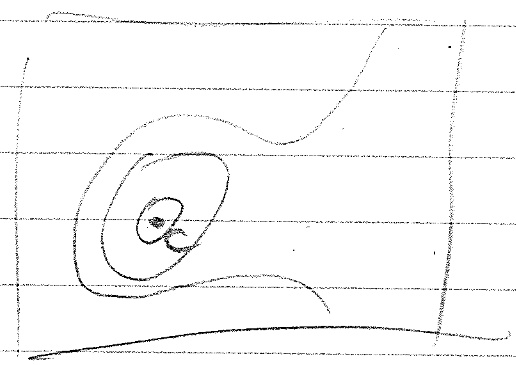
Problem B:

1) $f_x(A) > 0$ $\quad f_x(B) > 0$

$f_y(A) < 0$ $\quad f_y(B) = 0$.

2) eg point in the middle of

the 0.4 closed level curve



Problem C

$$r(b,c) = -\frac{b}{2} + \frac{\sqrt{b^2-4c}}{2}$$

Use linearisation:

$$\Delta r = L(b,c) - r(b,c) = r_b \, \Delta b + r_c \, \Delta c$$

Let $(p,q) = (-6,8)$. Then, $r(-6,8) = 4$

$(-6)^2 - 4 \cdot 8 = 4$

$$r_b = -\frac{1}{2} + \frac{1}{4} \frac{2b}{\sqrt{b^2-4c}}$$

$$= -\frac{1}{2} + \frac{b}{2\sqrt{b^2-4c}}$$

$$r_c = \frac{-4}{4\sqrt{b^2-4c}} = \frac{-1}{\sqrt{b^2-4c}}$$

$$\Rightarrow r_b(-6,8) = -\frac{1}{2} \quad \frac{-6}{2 \cdot 2} = -2$$

$$r_c(-6,8) = -\frac{1}{2}$$

$$L(-6.01, 7.98) = 4 + (-2)\cdot(-0.01) + (-\tfrac{1}{2})\cdot(-0.02)$$

$$= \underline{\underline{4.03}}$$

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<u>Problem A:</u>

1a) $f(x,y) = x^3 - 3x^2 + 2x + xy - x - y^2 + y$
$= x^3 - 3x^2 + x + xy - y^2 + y$

$$\nabla f = \begin{bmatrix} 3x^2 - 6x + 1 + y & +x - 2y + 1 \end{bmatrix}$$

$$\underline{r}'(t) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$D(f \circ \underline{r})(0) = \nabla f(\underline{r}(0))\, \underline{r}'(0)$$
$$= \begin{bmatrix} 2 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = 2 > 0$$

b) > 0 because

$$D(f \circ \underline{r})(0) = \frac{\partial f}{\partial x}(A)$$

<u>ie</u> AS MOVE W TO E ALONG $y = 1$, WE ARE GOING <u>UPHILL</u> AT A.

2) a) $D(f \circ \underline{s})(t)$

$$= \nabla f(\underline{s}(t))\, \underline{s}'(t)$$

$$= \begin{bmatrix} 3t^2 - 6t + 1 + 2t + 1 & t - 2(2t+1) + 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$= 3t^2 - 6t + 1 + (2t+1) + 2(t - 2(2t+1) + 1)$$

$$= 3t^2 - 10t = t(3t - 10)$$

b) Critical pts when $D(f \circ \underline{s})(t) = 0$:

ie $t = 0, \frac{10}{3}$

At $t = 0$; $f(\underline{s}(0)) = f(0,1) = 0$

$t = \frac{10}{3}$; $f(\underline{s}(\frac{10}{3})) = f(\frac{10}{3}, \frac{23}{3})$

$$= \frac{10}{3}\left(\frac{7}{3}\right)\left(\frac{4}{3}\right) + \left(\frac{20}{3}\right)\left(\frac{-13}{3}\right)$$

$$= \frac{280}{3^3} - \frac{260}{3^2}$$

$$= \frac{280 - 780}{3^3} = \frac{-500}{27}.$$

Also: at end points

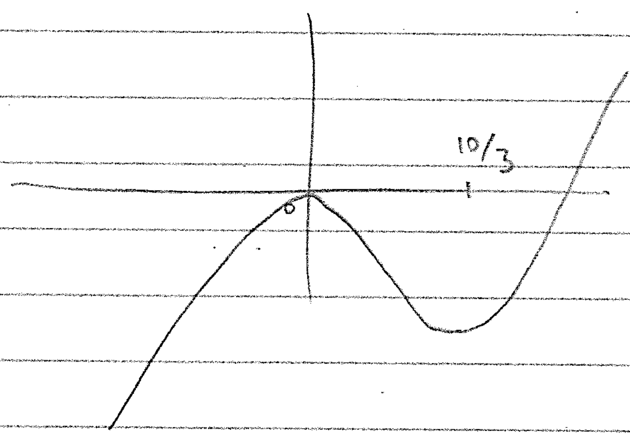
$t = -3$: $f(\underline{s}(-3)) = f(-3, -5) = -72$

$t = 6$: $f(\underline{s}(6)) = f(6, 13) = 36$

$\Rightarrow$ Highest elevation is 36 ;
lowest elevation is -72

c) Possibly ; will be walking uphill all morning

d) $(f \circ \underline{s})(t)$ models elevation exactly



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Problem A: Use : $\frac{\partial u}{\partial y} \neq \frac{\partial v}{\partial x} \Rightarrow$ NOT CONS.

1) $\frac{\partial u}{\partial y} = 0$

$\frac{\partial v}{\partial x} = e^{xy} + xy\,e^{xy}$

NOT CONSERVATIVE

2) $\frac{\partial u}{\partial y} = \cos(xy) \cdot x$

$\frac{\partial v}{\partial x} = \sin(xy) \cdot y$

NOT CONSERVATIVE

3) NOT CONSERVATIVE.